

1. ЗАДАЦИ

a) $D = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix} \cdot 10^{-6}$ $\vec{n} = \begin{bmatrix} 0.3333 \\ 0.6667 \\ 0.6667 \end{bmatrix}$ $E_n = \vec{n}^T D \vec{n} = 3.7778 \cdot 10^{-6}$
 $E_n = \vec{n}^T D \vec{n}$

$\begin{bmatrix} 0.3333 & 0.6667 & 0.6667 \end{bmatrix} \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix} \cdot 10^{-6} \cdot \begin{bmatrix} 0.3333 \\ 0.6667 \\ 0.6667 \end{bmatrix} = \begin{bmatrix} 2.6667 & 2.0001 & 2.3333 \end{bmatrix} \cdot 10^{-6}$

$\cdot \begin{bmatrix} 0.3333 \\ 0.6667 \\ 0.6667 \end{bmatrix} = 3.7778 \cdot 10^{-6}$

b) $D = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix} \cdot 10^{-6}$ $\vec{n} = \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix}$ $E_n = 0.5 \cdot 10^{-6}$

$\begin{bmatrix} 0.7071 & -0.7071 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix} \cdot 10^{-6} \cdot \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.4142 & 0.7071 & 0 \end{bmatrix} \cdot 10^{-6}$

$\cdot \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} = 0.5 \cdot 10^{-6}$

b) $D = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 5 & 0 \\ 2 & 0 & 3 \end{bmatrix} \cdot 10^{-6}$ $\vec{n} = \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix}$ $E_n = 3.6673 \cdot 10^{-6}$

$\begin{bmatrix} 0.5774 & 0.5774 & 0.5774 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 4 & 5 & 0 \\ 2 & 0 & 3 \end{bmatrix} \cdot 10^{-6} \cdot \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix} = \begin{bmatrix} 4.0418 & 4.0418 & 2.286 \end{bmatrix} \cdot 10^{-6}$

$\cdot \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix} = 3.6673 \cdot 10^{-6}$

ЗАДАЧА

$$D = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix} \cdot 10^{-6} \quad \vec{n} = \begin{bmatrix} 0.6667 \\ 0.6667 \\ 0.3333 \end{bmatrix} \quad \vec{m} = \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} \quad \frac{1}{2} \vec{g} \cdot \vec{m} \cdot \vec{n} = \vec{n}^T \vec{D} \vec{m}$$

$$\begin{bmatrix} 0.6667 & 0.6667 & 0.3333 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.0001 & 1.6666 & 2.3334 \end{bmatrix} \cdot 10^{-6}$$

$$\begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} = 0.2358 \cdot 10^{-6} = 8 \text{ nm} / 2$$

$8 \text{ nm} =$

$$D = \begin{bmatrix} 1 & 4 & 2 \\ 4 & -1 & 3 \\ 2 & 3 & 0 \end{bmatrix} \cdot 10^{-6} \quad \vec{n} = \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix} \quad \vec{m} = \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5774 & 0.5774 & 0.5774 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 2 \\ 4 & -1 & 3 \\ 2 & 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.0418 & 3.4644 & 2.8777 \end{bmatrix} \cdot 10^{-6}$$

$$\begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix} = 0.4083 \cdot 10^{-6} = 8 \text{ nm} = 8 \text{ nm} / 2$$

$8 \text{ nm} =$

ЗАДАЧА

$$D = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 27 \end{bmatrix} \cdot 10^{-6} \quad \det D = 0 \quad \begin{aligned} J_1 &= 3+12+27=42 \\ J_2 &= 0+0+0=0 \\ J_3 &= 0 \end{aligned}$$

линейно связанные
 $\Delta E \text{ доминирует}$
 $E^3 - 42E^2 = 0$

$$D = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 12 & 18 \\ 9 & 18 & 3 \end{bmatrix} \cdot 10^{-6} \quad \det D = 0 \quad \begin{aligned} J_1 &= 3+12+3=18 \\ J_2 &= -288-72+0=-360 \\ J_3 &= 0 \end{aligned}$$

равно связанные
 $\Delta E \text{ доминирует}$

$$E^3 - 18E^2 - 360E = 0$$

$B) D = \begin{bmatrix} 3 & 6 & 0 \\ 6 & 12 & 18 \\ 0 & 18 & 3 \end{bmatrix} \cdot 10^{-6}$
 $\det D = -972$
 $f_1 = 3 + 12 + 3 = 18$
 $f_2 = -288 + 9 + 0 = -279$
 $f_3 = -972$

$\begin{bmatrix} 3 & 6 & 0 \\ 6 & 12 & 18 \\ 0 & 18 & 3 \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 6 & 12 \\ 0 & 18 \end{bmatrix} = 108 - 972 - 108 = -972$
 $E^3 - 18E^2 - 279E + 972 = 0$

$F) D = \begin{bmatrix} 3 & 2 & 9 \\ 2 & 12 & 6 \\ 9 & 6 & 3 \end{bmatrix} \cdot 10^{-6}$
 $\det D = -768$
 $f_1 = 3 + 12 + 3 = 18$
 $f_2 = 0 + 72 + 32 = -40$
 $f_3 = -768$

$\begin{bmatrix} 3 & 2 & 9 \\ 2 & 12 & 6 \\ 9 & 6 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 12 \\ 9 & 6 \end{bmatrix} = 108 + 108 + 108 - 972 - 108 - 12 = -768$
 $E^3 - 18E^2 + 40E + 768 = 0$

4. ЗАДАЧА

$R(x^0, y^0, z^0, t) = \begin{cases} x^0 + (x^0)^2 t \\ y^0 + 2(y^0 z^0)^2 t \\ z^0 + 3t \end{cases}$

$A) \vec{S} = \begin{cases} (x^0)^2 t = u \\ 2(y^0 z^0)^2 t = v \\ 3t = w \end{cases}$

$B) \frac{du}{dx} = 2x^0 t \quad \frac{dv}{dx} = 0 \quad \frac{dw}{dx} = 0$
 $\frac{du}{dy} = 0 \quad \frac{dv}{dy} = 4y^0 z^0 t^2 \quad \frac{dw}{dy} = 0$
 $\frac{du}{dz} = 0 \quad \frac{dv}{dz} = 4(y^0)^2 z^0 t^2 \quad \frac{dw}{dz} = 0$

$B) D = \begin{bmatrix} 2x^0 t & \frac{1}{2}(0+0) & \frac{1}{2}(0+0) \\ 0 & 4y^0(z^0)^2 & \frac{1}{2}(4y^0 z^0 t^2 + 0) \\ 0 & \frac{1}{2}(4(y^0)^2 z^0 t^2 + 0) & 0 \end{bmatrix} = \begin{bmatrix} 2t & 0 & 0 \\ 0 & 36 & 6 \\ 0 & 6 & 0 \end{bmatrix}$

$R(1,1,3)$

$A) E_n = n^T D_n = n^T \begin{bmatrix} 2t & 0 & 0 \\ 0 & 12 & 18 \\ 0 & 18 & 0 \end{bmatrix} \cdot n$
 $R(1,3,1)$

3

35

$$5) \mathcal{E}^{ns} = \eta^T \mathcal{D} S$$

$$E) \mathcal{E} = \mathcal{E}_x + \mathcal{E}_y + \mathcal{E}_z = 2x^2 z + 4yz(z-1)^2$$

6. ЗАДАЧА

$$\mathcal{D} = \begin{bmatrix} A \times y & x \times z & 2yz \\ x \times z & -2(y-1)^3 & x \times z \\ 2yz & x \times z & A \times y \end{bmatrix} \cdot 10^{-6}$$

Saint-Venant's

уравнения совместности деформаций

Δε деформации

$$\frac{\partial^2 \mathcal{E}_x}{\partial y^2} + \frac{\partial^2 \mathcal{E}_y}{\partial x^2} = \frac{\partial^2 \mathcal{E}_{xy}}{\partial x \partial y} \quad (1)$$

$$\frac{\partial^2 \mathcal{E}_x}{\partial z^2} + \frac{\partial^2 \mathcal{E}_z}{\partial x^2} = \frac{\partial^2 \mathcal{E}_{xz}}{\partial x \partial z} \quad (2)$$

$$\frac{\partial^2 \mathcal{E}_y}{\partial z^2} + \frac{\partial^2 \mathcal{E}_z}{\partial y^2} = \frac{\partial^2 \mathcal{E}_{yz}}{\partial y \partial z} \quad (3)$$

$$\frac{\partial^2 \mathcal{E}_x}{\partial y \partial z} = \frac{1}{2} \frac{\partial}{\partial x} \left(-\frac{\partial^2 xz}{\partial y^2} + \frac{\partial^2 xz}{\partial x^2} - \frac{\partial^2 xz}{\partial z^2} \right) \quad (4)$$

$$\frac{\partial^2 \mathcal{E}_y}{\partial x \partial z} = \frac{1}{2} \frac{\partial}{\partial y} \left(-\frac{\partial^2 xz}{\partial y^2} + \frac{\partial^2 xz}{\partial x^2} + \frac{\partial^2 xz}{\partial z^2} \right)$$

$$\frac{\partial^2 \mathcal{E}_z}{\partial y \partial x} = \frac{1}{2} \frac{\partial}{\partial z} \left(-\frac{\partial^2 xz}{\partial y^2} + \frac{\partial^2 xz}{\partial x^2} + \frac{\partial^2 xz}{\partial y^2} \right)$$

$$(1) \frac{\partial}{\partial y} (Ax) + \frac{\partial}{\partial x} (0) = \frac{\partial}{\partial x} (2xz)$$

$$0 + 0 = 2z \quad 2z = 0 \quad \boxed{z = 0} \quad \checkmark$$

$$(2) \frac{\partial}{\partial z} (0) + \frac{\partial}{\partial x} (Ay) = \frac{\partial}{\partial x} (0)$$

$$0 + 0 = 0 \quad \boxed{0 = 0} \quad \checkmark$$

$$(3) \frac{\partial}{\partial z} (0) + \frac{\partial}{\partial y} (Ax) = \frac{\partial}{\partial z} (2xz)$$

$$0 + 0 = 2x - 2x = 0 \quad \boxed{x = 0} \quad \checkmark$$

$$(4) \frac{\partial}{\partial y} (0) = \frac{1}{2} \frac{\partial}{\partial x} (-2yz + 2\mathcal{D}z + 2xy)$$

$$0 = \frac{1}{2} (0 + 0 + 2y)$$

$$0 = \frac{1}{2} \cdot 2y \quad \boxed{y = 0} \quad \checkmark$$

$$(5) \frac{\partial}{\partial x} (0) = \frac{1}{2} \frac{\partial}{\partial y} (-2\mathcal{D}z + 2yz + 2xy)$$

$$0 = \frac{1}{2} (0 + 2z + 2x)$$

$$0 = z + x \quad x + z = 0 \quad \boxed{z = -x} \quad \boxed{x = 0} \quad \boxed{z = 0} \quad \checkmark$$

$$(6) \frac{\partial}{\partial x} (Ax) = \frac{1}{2} \frac{\partial}{\partial z} (-2xy + 2yz + 2\mathcal{D}z)$$

$$A = \frac{1}{2} (0 + 2y + 2\mathcal{D})$$

$$A = y + \mathcal{D} \quad \boxed{y = A - \mathcal{D}} \quad \checkmark$$

$$y = 0 \quad A - \mathcal{D} = 0$$

$$A = \mathcal{D}$$

ЗАДАЧА

б) $u(x,y) = Ax + By + C$ $V(x,y) = Ax + By + C$ $\mathcal{D} = \begin{bmatrix} 1/6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\vec{n} = \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix}$

$u(0,0) = 0 = C$ $C = 0$ $V(0,0) = C = 0$ $C = 0$

$u(6,0) = 6A = 1$ $A = 1/6$ $V(6,0) = 6A = 0$ $A = 0$

$u(0,6) = 6B = 0$ $B = 0$ $V(0,6) = 6B = 0$ $B = 0$ $E = 1/6$

$u = \frac{x}{6}$ $\frac{\partial u}{\partial x} = \frac{1}{6}$ $\frac{\partial u}{\partial y} = 0$ $\frac{\partial u}{\partial z} = 0$ $u = 0$ $\frac{\partial u}{\partial x} = 0$ $\frac{\partial u}{\partial y} = 0$ $\frac{\partial u}{\partial z} = 0$

$E_n = \vec{n} \cdot \mathcal{D} \vec{n} = \begin{Bmatrix} 0.866 & 0.5 & 0 \end{Bmatrix} \cdot \begin{bmatrix} 1/6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.1443 & 0 & 0 \end{Bmatrix} \cdot \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix}$

$E_n = 0.1250$

б) $u(x,y) = Ax + By + C$ $V(x,y) = Ax + By + C$ $\mathcal{D} = \begin{bmatrix} 1/6 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\vec{n} = \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix}$

$u(0,0) = 0 = C$ $C = 0$ $V(0,0) = C = 0$ $C = 0$

$u(6,0) = 6A = 1$ $A = 1/6$ $V(6,0) = 6A = 0$ $A = 0$

$u(0,6) = 6B = 0$ $B = 0$ $V(0,6) = 6B = -1$ $B = -1/6$ $E = 0$

$u = x/6$ $\frac{\partial u}{\partial x} = \frac{1}{6}$ $\frac{\partial u}{\partial y} = 0$ $\frac{\partial u}{\partial z} = 0$ $V = -y/6$ $\frac{\partial V}{\partial x} = 0$ $\frac{\partial V}{\partial y} = -1/6$ $\frac{\partial V}{\partial z} = 0$

$E = \begin{Bmatrix} 0.866 & 0.5 & 0 \end{Bmatrix} \cdot \begin{bmatrix} 1/6 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.1443 & 0.0833 & 0 \end{Bmatrix} \cdot \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix}$

$E_n = 0.1666$

г) $u(x,y) = Ax + By + C$ $V(x,y) = Ax + By + C$ $\mathcal{D} = \begin{bmatrix} 0 & 1/6 & 0 \\ 1/6 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\vec{n} = \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix}$

$u(0,0) = 0 = C$ $C = 0$ $V(0,0) = C = 0$ $C = 0$

$u(6,0) = 6A = 0$ $A = 0$ $V(6,0) = 6A = 1$ $A = 1/6$

$u(0,6) = 6B = 1$ $B = 1/6$ $V(0,6) = 6B = 0$ $B = 0$

$u = y/6$ $\frac{\partial u}{\partial x} = 0$ $\frac{\partial u}{\partial y} = \frac{1}{6}$ $\frac{\partial u}{\partial z} = 0$ $V = x/6$ $\frac{\partial V}{\partial x} = \frac{1}{6}$ $\frac{\partial V}{\partial y} = 0$ $\frac{\partial V}{\partial z} = 0$

$E_n = \begin{Bmatrix} 0.866 & 0.5 & 0 \end{Bmatrix} \cdot \begin{bmatrix} 0 & 1/6 & 0 \\ 1/6 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.0833 & 0.1443 & 0 \end{Bmatrix} \cdot \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix}$

$E_n = 0.1443$

$$1) u(x,y) = Ax + By + C$$

$$v(x,y) = Ax + By + C$$

$$\vec{n} = \begin{bmatrix} 0 & 1/12 & 0 \end{bmatrix} \quad \vec{n} = \begin{bmatrix} 0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$u(0,0) = C = 0 \quad \left\{ \begin{array}{l} C=0 \end{array} \right.$$

$$v(0,0) = C = 0 \quad \left\{ \begin{array}{l} C=0 \end{array} \right.$$

$$\begin{bmatrix} 1/12 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$u(6,0) = 6A = 1 \quad \left\{ \begin{array}{l} A = 1/6 \end{array} \right.$$

$$v(6,0) = 6A = 0 \quad \left\{ \begin{array}{l} A = 0 \end{array} \right.$$

$$u(0,6) = 6B = 0 \quad \left\{ \begin{array}{l} B = 0 \end{array} \right.$$

$$v(0,6) = 6B = 0 \quad \left\{ \begin{array}{l} B = 0 \end{array} \right.$$

$$u = \frac{y}{6} \quad \frac{\partial u}{\partial x} = 0 = \frac{\partial u}{\partial z} \quad \frac{\partial u}{\partial y} = 1/6 \quad v = 0 \quad \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial v}{\partial z} = 0$$

$$\vec{E}_n = \begin{bmatrix} 0.866 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1/12 & 0 \\ 1/12 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.866 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0417 & 0.0722 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.866 \\ 0.5 \\ 0 \end{bmatrix} =$$

$$\vec{E}_n = 0.0722$$

$$5) u(x,y) = Ax + By + C$$

$$v(x,y) = Ax + By + C$$

$$\vec{n} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \vec{n} = \begin{bmatrix} 0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$u(0,0) = C = 0 \quad \left\{ \begin{array}{l} C=0 \end{array} \right.$$

$$v(0,0) = C = 0 \quad \left\{ \begin{array}{l} C=0 \end{array} \right.$$

$$\vec{n} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$u(6,0) = 6A = 0 \quad \left\{ \begin{array}{l} A = 0 \end{array} \right.$$

$$v(6,0) = 6A = 1 \quad \left\{ \begin{array}{l} A = 1/6 \end{array} \right.$$

$$u(0,6) = 6B = -1 \quad \left\{ \begin{array}{l} B = -1/6 \end{array} \right.$$

$$v(0,6) = 6B = 0 \quad \left\{ \begin{array}{l} B = 0 \end{array} \right.$$

$$u = -\frac{y}{6} \quad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} = 0 \quad v = x/6 \quad \frac{\partial v}{\partial x} = \frac{1}{6} \quad \frac{\partial v}{\partial y} = \frac{\partial v}{\partial z} = 0$$

$$\frac{\partial u}{\partial y} = -1/6$$

$$\vec{E}_n = \begin{bmatrix} 0.866 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0.866 \\ 0.5 \\ 0 \end{bmatrix} = 0$$

КОНСТИТУТИВНЕ ВЕЗЕ

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu (\sigma_y + \sigma_z)) & \frac{1}{2} \gamma_{xy} &= \frac{1+\nu}{E} \tau_{xy} = \frac{1}{2G} \tau_{xy} \\ \epsilon_y &= \frac{1}{E} (\sigma_y - \nu (\sigma_x + \sigma_z)) & \frac{1}{2} \gamma_{xz} &= \frac{1+\nu}{E} \tau_{xz} = \frac{1}{2G} \tau_{xz} \\ \epsilon_z &= \frac{1}{E} (\sigma_z - \nu (\sigma_x + \sigma_y)) & \frac{1}{2} \gamma_{zy} &= \frac{1+\nu}{E} \tau_{zy} = \frac{1}{2G} \tau_{zy} \end{aligned}$$

$$G = \frac{E}{2(1+\nu)} \quad K = \frac{E}{3(1-2\nu)} \quad e = \frac{\sigma}{K}$$

$$\mu = G = \frac{E}{2(1+\nu)}$$

$$\lambda = \frac{\nu E}{(1-2\nu)(1+\nu)}$$

$$\begin{aligned} \sigma_x &= 2\mu \epsilon_x + \lambda e & \tau_{xy} &= \mu \gamma_{xy} \\ \sigma_y &= 2\mu \epsilon_y + \lambda e & \tau_{xz} &= \mu \gamma_{xz} \\ \sigma_z &= 2\mu \epsilon_z + \lambda e & \tau_{yz} &= \mu \gamma_{yz} \end{aligned}$$

ОПОРНОСТЬ МАТЕРИАЛА: 1 ЗАДАЧИ ЗА 2 КОЛОКВИУМ ~ КОНСТИТУТИВНЕ БЕЗЕ ~

1. ЗАДАЧА

A) $S = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot 10^6$ $E = 210 \text{ GPa}$ $\bar{\nu} = \begin{Bmatrix} 0.7071 \\ 0.7071 \\ 0 \end{Bmatrix}$ $\mu = 84 \text{ GPa}$
 $\nu = 0.25$ $\lambda = 84 \text{ GPa}$ $E = 4$

$\epsilon_x = 2$ $\gamma_{xy} = 0$ $\sigma_x = 2 \cdot 84 \cdot 2 + 84 \cdot 4 = 672$ $\tau_{xy} = 84 \cdot 0 = 0$
 $\epsilon_y = 2$ $\gamma_{xz} = 2$ $\sigma_y = 2 \cdot 84 \cdot 2 + 84 \cdot 4 = 672$ $\tau_{yz} = 84 \cdot 2 = 168$
 $\epsilon_z = 0$ $\gamma_{yz} = 2$ $\sigma_z = 2 \cdot 84 \cdot 0 + 84 \cdot 4 = 336$ $\tau_{yz} = 84 \cdot 2 = 168$

$S = \begin{bmatrix} 672 & 0 & 168 \\ 0 & 672 & 168 \\ 168 & 168 & 336 \end{bmatrix} \text{ kPa}$ $\bar{\sigma}^{(m)} = \begin{Bmatrix} 672 \cdot 0.7071 + 0 \cdot 0.7071 + 0 \cdot 168 \\ 672 \cdot 0.7071 \\ 0.7071 \cdot 168 \cdot 2 \end{Bmatrix} = \begin{Bmatrix} 475.1712 \\ 475.1712 \\ 237.5856 \end{Bmatrix} \text{ kPa}$

B) $S = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot 10^{-6}$ $\bar{\nu} = \begin{Bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{Bmatrix}$ $E = 200 \text{ GPa}$ $\lambda = 55.5556 \text{ GPa}$
 $\nu = 0.2$ $\mu = 83.3333 \text{ GPa}$ $E = 2$

$\epsilon_x = 2$ $\gamma_{xz} = 0$ $\sigma_x = 2 \cdot 83.3333 \cdot 2 + 55.5556 \cdot 2 = 444.4444$
 $\epsilon_y = 0$ $\gamma_{xy} = 4$ $\sigma_y = 2 \cdot 83.3333 \cdot 0 + 55.5556 \cdot 2 = 111.1112$
 $\epsilon_z = 0$ $\gamma_{yz} = 2$ $\sigma_z = 2 \cdot 83.3333 \cdot 0 + 55.5556 \cdot 2 = 111.1112$

$\tau_{xy} = 83.3333 \cdot 0 = 0$ $\tau_{yz} = 83.3333 \cdot 4 = 333.3332$ $\tau_{xz} = 83.3333 \cdot 2 = 166.6666$
 $S = \begin{bmatrix} 444.4444 & 0 & 333.3332 \\ 0 & 111.1112 & 166.6666 \\ 333.3332 & 166.6666 & 111.1112 \end{bmatrix} \text{ kPa}$

$\bar{\sigma}^{(m)} = \begin{Bmatrix} 449.0888 \\ 160.3889 \\ 332.8555 \end{Bmatrix} \text{ kPa}$

2. ЗАДАЧА

A) $S = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix} \text{ MPa}$ $\bar{\nu} = \begin{Bmatrix} 0.5774 \\ 0.3464 \\ 0.4619 \end{Bmatrix}$ $E = 70 \text{ GPa}$ $\nu = 0.15$
 $\nu = 0.15$ $\mu = 24.5 \text{ GPa}$ $E = 10^3$

$\epsilon_x = 3$ $\tau_{xy} = 0$ $\epsilon_y = 1$ $\tau_{xz} = 2$ $\epsilon_z = -1$ $\tau_{yz} = 0$
 $\sigma_x = \frac{1}{70} (3 - 0.15(1-1)) = 42.8571$
 $\sigma_y = \frac{1}{70} (1 - 0.15(3-1)) = 10$
 $\sigma_z = \frac{1}{70} (-1 - 0.15(3+1)) = -22.8571$

$$\frac{1}{2} \sigma_{xy} = \frac{1+0.15}{70} \cdot 0 = 0$$

$$\frac{1}{2} \sigma_{xz} = \frac{1+0.15}{70} \cdot 2 = 32.8571 \cdot 10^{-6}$$

$$\frac{1}{2} \sigma_{yz} = \frac{1+0.15}{70} \cdot 0 = 0$$

$$\sigma = \begin{bmatrix} 42.8571 & 0 & 32.8571 \\ 0 & 10 & 0 \\ 32.8571 & 0 & -22.8571 \end{bmatrix} \cdot 10^{-6}$$

$$\epsilon_n = \begin{bmatrix} 0.5774 & 0.3464 & 0.4619 \end{bmatrix} \begin{bmatrix} 42.8571 & 0 & 32.8571 \\ 0 & 10 & 0 \\ 32.8571 & 0 & -22.8571 \end{bmatrix} \cdot 10^{-6} \cdot \begin{Bmatrix} 0.5774 \\ 0.3464 \\ 0.4619 \end{Bmatrix}$$

$$\epsilon_n = \begin{Bmatrix} 39.9224 & 3.464 & 8.4140 \end{Bmatrix} \cdot 10^{-6} \cdot \begin{Bmatrix} 0.5774 \\ 0.3464 \\ 0.4619 \end{Bmatrix} = 28.1375 \cdot 10^{-6}$$

$$b) S = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \cdot 10^6 \quad \bar{n} = \begin{Bmatrix} 0.3464 \\ 0 \\ 0.4619 \end{Bmatrix} \quad E = 80 \text{ GPa} \\ \nu = 0.25$$

$$\sigma_x = -2 \quad \tau_{xy} = 0 \quad \epsilon_x = \frac{1}{80} (-2 - 0.25(1-1)) = -25 \cdot 10^{-6}$$

$$\sigma_y = 1 \quad \tau_{yz} = 0 \quad \epsilon_y = \frac{1}{80} (1 - 0.25(-2-1)) = 21.875 \cdot 10^{-6}$$

$$\sigma_z = -1 \quad \tau_{yz} = 1 \quad \epsilon_z = \frac{1}{80} (-1 - 0.25(-2+1)) = -9.375 \cdot 10^{-6}$$

$$\frac{1}{2} \sigma_{xy} = \frac{1+0.25}{80} \cdot 0 = 0$$

$$\frac{1}{2} \sigma_{xz} = \frac{1+0.25}{80} \cdot 0 = 0$$

$$\frac{1}{2} \sigma_{yz} = \frac{1+0.25}{80} \cdot 1 = 15.625 \cdot 10^{-6}$$

$$\sigma = \begin{bmatrix} -25 & 0 & 0 \\ 0 & 21.875 & 15.625 \\ 0 & 15.625 & -9.375 \end{bmatrix} \cdot 10^{-6}$$

$$\epsilon_n = \begin{bmatrix} 0.3464 & 0 & 0.4619 \end{bmatrix} \cdot 10^{-6} \begin{bmatrix} -25 & 0 & 0 \\ 0 & 21.875 & 15.625 \\ 0 & 15.625 & -9.375 \end{bmatrix} \cdot \begin{Bmatrix} 0.3464 \\ 0 \\ 0.4619 \end{Bmatrix}$$

$$\epsilon_n = \begin{Bmatrix} -8.66 & 7.2172 & 4.3303 \end{Bmatrix} \cdot \begin{Bmatrix} 0.3464 \\ 0 \\ 0.4619 \end{Bmatrix} = -0.9996$$

3. ЗАДАЧА

$$S = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad \begin{matrix} \epsilon_x = 3 \cdot 10^{-6} \\ \epsilon_y = -1 \cdot 10^{-6} \\ \bar{\epsilon} = 2 \end{matrix} \quad \begin{matrix} G_x = 4 \\ G_y = 1 \\ G_z = 1 \end{matrix} \quad \begin{matrix} \tau_{xy} = 0 \\ \tau_{yz} = 0 \\ \tau_{xz} = 2 \end{matrix} \quad \left. \vphantom{\begin{matrix} \epsilon_x \\ \epsilon_y \\ \bar{\epsilon} \end{matrix}} \right\} \begin{matrix} \epsilon = (2 + \epsilon_z) \cdot 10^{-6} \\ \epsilon = 10^{-6} \end{matrix}$$

$$3 \cdot 10^{-6} = \frac{1}{E} (4 - \nu(1+1)) \cdot 10^6 \quad K = 2004.1338 \text{ GPa}$$

$$-1 \cdot 10^{-6} = \frac{1}{E} (1 - \nu(4+1)) \cdot 10^6$$

$$3 = \frac{10^{12}}{E} (4 - 2\nu) \quad -1 = \frac{10^{12}}{E} (1 - 5\nu)$$

$$-1 = \frac{10^{12}}{E} (1 - 5\nu) \quad E = 1059 \text{ GPa}$$

$$-3 = \frac{4 - 2\nu}{1 - 5\nu} \quad \epsilon_z = \frac{1}{1059} (1 - 0.4118(4+1)) = -1 \cdot 10^{-6}$$

$$\frac{1}{2} \sigma_{xy} = \frac{1 + 0.4118}{1059} \cdot 0 = 0$$

$$\frac{1}{2} \sigma_{xz} = \frac{1 + 0.4118}{1059} \cdot 2 = 2.663 \cdot 10^{-6}$$

$$\frac{1}{2} \sigma_{yz} = \frac{1 + 0.4118}{1059} \cdot 0 = 0$$

4. ЗАДАЧА

$$\epsilon_{n1} = 20 \cdot 10^{-6} \quad 20^\circ \quad 40^\circ = 2\alpha_1$$

$$\epsilon_{n2} = 5 \cdot 10^{-6} \quad 65^\circ \quad 130^\circ = 2\alpha_2$$

$$\epsilon_{n3} = -15 \cdot 10^{-6} \quad 110^\circ \quad 220^\circ = 2\alpha_3$$

$$A) \quad 20 \cdot 10^{-6} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 40^\circ + \frac{1}{2} \sigma_{xy} \sin 40^\circ / 2$$

$$5 \cdot 10^{-6} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 130^\circ + \frac{1}{2} \sigma_{xy} \sin 130^\circ / 2$$

$$-15 \cdot 10^{-6} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 220^\circ + \frac{1}{2} \sigma_{xy} \sin 220^\circ / 2$$

$$40 = \epsilon_x + \epsilon_y + 0.766 \epsilon_x - 0.766 \epsilon_y + 0.6428 \sigma_{xy}$$

$$10 = \epsilon_x + \epsilon_y - 0.6428 \epsilon_x + 0.6428 \epsilon_y + 0.766 \sigma_{xy}$$

$$-30 = \epsilon_x + \epsilon_y - 0.766 \epsilon_x + 0.766 \epsilon_y - 0.6428 \sigma_{xy}$$

$$1.766 \epsilon_x + 0.234 \epsilon_y + 0.6428 \sigma_{xy} = 40$$

$$0.3572 \epsilon_x + 1.6428 \epsilon_y + 0.766 \sigma_{xy} = 10$$

$$0.234 \epsilon_x + 1.766 \epsilon_y - 0.6428 \sigma_{xy} = -30$$

$$\epsilon_x = 14.2986 \cdot 10^{-6}$$

$$\epsilon_y = -9.2986 \cdot 10^{-6}$$

$$\sigma_{xy} = 26.3294 \cdot 10^{-6}$$

11

5

11

$$5) E_{1/2} = \frac{E_x + E_y}{2} \pm \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{8xy}{2}\right)^2}$$

$$E_{1/2} = \frac{14.2586 + 9.2586}{2} \pm \sqrt{\left(\frac{14.2586 - 9.2586}{2}\right)^2 + \left(\frac{26.3251}{2}\right)^2}$$

$$E_{1/2} = 2.5 \pm 17.6781$$

$$E_1 = 20.1781 \cdot 10^{-6}$$

$$E_2 = -15.1781 \cdot 10^{-6}$$

$$B) G_z = 2\mu E_z + \lambda e = 0$$

$$2 \cdot \frac{E}{2(1+\gamma)} \cdot E_z + \frac{\gamma E}{(1+\gamma)(1-2\gamma)} = (E_x + E_y + E_z) = 0$$

$$\frac{E}{1+\gamma} E_z + \frac{\gamma E}{(1+\gamma)(1-2\gamma)} \cdot (14.2586 - 9.2586 + E_z) = 0$$

$$\frac{E}{1+\gamma} E_z + \frac{\gamma E}{(1+\gamma)(1-2\gamma)} \cdot (2.5 + E_z) = 0$$

$$\frac{E}{1+\gamma} E_z + \frac{\gamma E}{(1+\gamma)(1-2\gamma)} \cdot 2.5 + \frac{\gamma E}{(1+\gamma)(1-2\gamma)} E_z = 0$$

$$\frac{E(1-2\gamma) + \gamma E}{(1+\gamma)(1-2\gamma)} E_z = -\frac{2.5\gamma E}{(1+\gamma)(1-2\gamma)}$$

$$E_z = \frac{-2.5\gamma E}{(1+\gamma)(1-2\gamma)} \cdot \frac{(1+\gamma)(1-2\gamma)}{E - 2\gamma E + \gamma E} = \frac{-2.5\gamma E}{E - \gamma E} = \frac{-2.5\gamma E}{E(1-\gamma)}$$

$$\frac{1}{E_z} = -\frac{2.5\gamma}{1-\gamma} = \frac{2.5\gamma}{\gamma-1}$$

$$e = E_x + E_y + E_z = 14.2586 - 9.2586 + \frac{2.5\gamma}{\gamma-1} = 2.5 + \frac{2.5\gamma}{\gamma-1} = \frac{2.5\gamma - 2.5 + 2.5\gamma}{\gamma-1}$$

$$e = \frac{2.5(2\gamma-1)}{(\gamma-1)}$$

$$\Gamma) G_z = 0 \quad G_x = 2\mu E_x + \lambda e = A$$

$$A = 2 \cdot \frac{E}{2(1+\gamma)} \cdot 14.2586 + \frac{\gamma E}{(1+\gamma)(1-2\gamma)}$$

$$\tau_{xz} = 0 \quad G_y = 2\mu E_y + \lambda e = B$$

$$\tau_{yz} = 0 \quad \tau_{xy} = \mu 8xy = C$$

$$S = \begin{bmatrix} A & C & 0 \\ C & B & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \frac{14.2586 E}{(1+\gamma)} + \frac{2.5\gamma E}{1-\gamma^2}$$

$$A = \frac{14.2586 E - 14.2586 \gamma E + 2.5\gamma E}{1-\gamma^2}$$

$$12 \quad A = \frac{1}{1-\gamma^2} (14.2586 E - 14.2586 \gamma E + 2.5\gamma E) = \frac{E}{1-\gamma^2} (14.2586 - 14.2586 \gamma + 2.5\gamma)$$

$$B = 2 \cdot \frac{E}{2(1+\nu)} \cdot (-0.2586) + \frac{\nu E}{(1+\nu)(1-2\nu)} \cdot \frac{-2.5(1-2\nu)}{\nu-1}$$

$$B = \frac{-9.2586E}{1+\nu} + \frac{2.5\nu E}{1-\nu^2} = \frac{-9.2586E + 9.2586\nu E + 2.5\nu E}{1-\nu^2} =$$

$$= \frac{E}{(1-\nu^2)} (-9.2586 + 11.7986\nu)$$

$$C = \frac{E}{2(1+\nu)} \cdot 26.3294 = \frac{E}{1+\nu} \cdot 13.1647$$

5.3AAATAK

$$F = 20 \text{ kN}$$

$$\epsilon_x = \epsilon_y = \frac{1}{200} = 0.005 \quad \epsilon_z = \frac{-3}{200} = -0.015$$

$$a = 20 \text{ cm}$$

$$G_z = \frac{26.10}{20^2} = 0.5 \text{ MPa}$$

$$\epsilon_z = \frac{1}{E} (G_z + \nu(G_x + G_y))$$

$$\Delta \epsilon_z = -3 \text{ mm}$$

$$-0.015 = \frac{1}{E} (-0.5 \cdot 10^6 - \nu \cdot (0 + 0))$$

$$\Delta \epsilon_x = \Delta \epsilon_y = +1 \text{ mm}$$

$$-0.015 = -0.5 \cdot 10^6 / E$$

$$G_x = 0 \text{ MPa} = G_y$$

$$E = -0.5 \cdot 10^6 / -0.015$$

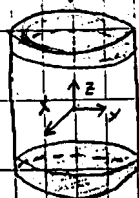
$$E = 33.3333 \text{ MPa}$$

$$\nu = \frac{-\epsilon_y}{\epsilon_z} = \frac{-0.005}{-0.015} = 0.3333 = 1/3$$

$$G = \frac{E}{2(1+\nu)} = \frac{33.3333 \text{ MPa}}{2(1+0.3333)} = 12.5003 \text{ MPa}$$

$$K = \frac{E}{3(1-2\nu)} = \frac{33.3333 \text{ MPa}}{3(1-2 \cdot 0.3333)} = 33.3266 \text{ MPa}$$

ЗАДАЧА



$$D = 20 \text{ cm}$$

$$H = 20 \text{ cm}$$

$$P = 15 \text{ kN}$$

$$A = \left(\frac{D}{2}\right)^2 \pi = \left(\frac{20}{2}\right)^2 \pi = 314,1593 \text{ cm}^2$$

$$\sigma_z = \frac{P_z}{A} = \frac{-15 \cdot 10}{314,1593} = -0,4775 \text{ MPa}$$

$$P_z = -15 \text{ kN}$$

$$\Delta e_z = -0,2 \text{ mm} \quad \sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xz} = 0 \quad \tau_{yz} = 0$$

$$\Delta e_x = \Delta e_y = 0,07 \text{ mm}$$

$$a) \epsilon_x = \epsilon_y = \frac{\Delta e_x}{e_x} = \frac{\Delta e_y}{e_y} = \frac{0,07}{200} = 3,5 \cdot 10^{-4}$$

$$\epsilon_z = \frac{\Delta e_z}{e_z} = \frac{-0,2}{200} = -0,001$$

$$\epsilon_z = \frac{1}{E} (\sigma_z - \nu (\sigma_x + \sigma_y)) \Rightarrow -0,001 = \frac{1}{E} (-0,4775 \cdot 10^6 - \cancel{\nu (0 + 0)}) \Rightarrow$$

$$-0,001 = - \frac{0,4775 \cdot 10^6}{E}$$

$$E = \frac{-0,4775 \cdot 10^6}{-0,001} = 477,5 \cdot 10^6 \text{ Pa} = 477,5 \text{ MPa}$$

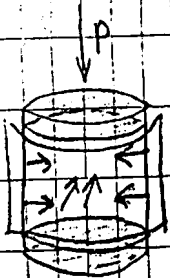
$$\nu = - \frac{\epsilon_y}{\epsilon_z} = \frac{-0,07}{-0,2} = 0,35$$

$$G = \frac{477,5}{2(1+0,35)} = 176,8518 \text{ MPa} = \mu$$

$$\lambda = \frac{0,35 \cdot 477,5}{1+0,35(1-2 \cdot 0,35)} = 412,6543 \text{ MPa}$$

$$K = \frac{477,5}{3(1-2 \cdot 0,35)} = 530,5556 \text{ MPa}$$

5)



$$D = 200 \text{ mm}$$

$$h = 200 \text{ mm}$$

$$E = 477.5 \text{ MPa}$$

$$\nu = 0.35$$

$$P = 15 \text{ kN}$$

$$P = -15 \text{ kN}$$

$$A = \left(\frac{D}{2}\right)^2 \bar{a} = \left(\frac{200}{2}\right)^2 \bar{a} = 314.1593 \text{ cm}^2$$

$$G_z = \frac{P}{A} = \frac{-15 \cdot 10}{314.1593} = -0.4775 \text{ MPa}$$

$$E_x = E_y = 0$$

$$G_y \neq 0 \quad G_x \neq 0 \quad \tau_{xy} = 0, \tau_{xz} = 0, \tau_{yx} = 0$$

$$E_x = \frac{1}{E} (G_x - \nu (G_y + G_z))$$

$$E_y = \frac{1}{E} (G_y - \nu (G_x + G_z))$$

$$0 = \frac{1}{477.5} (G_x - 0.35 (G_y - 0.4775))$$

$$E_z = \frac{\Delta \epsilon_z}{\epsilon_z}$$

$$0 = \frac{1}{477.5} (G_y - 0.35 (G_x - 0.4775))$$

$$\Delta \epsilon_z = \epsilon_z \cdot \epsilon_z$$

$$\epsilon_z = 200 \text{ mm} = 0.2 \text{ m}$$

$$\epsilon_z = -6.2310 \cdot 10^{-4}$$

$$G_x - 0.35 G_y + 0.1671 = 0$$

$$G_y - 0.35 G_x + 0.1671 = 0 \quad / \cdot \frac{1}{0.35} + \uparrow$$

$$\Delta \epsilon_z = -1.2464 \cdot 10^{-4} \text{ m}$$

$$\Delta \epsilon_z = -0.1246 \text{ mm}$$

$$G_x - 0.35 G_y + 0.1671 = 0$$

$$2.8571 G_y - G_x + 0.4774 = 0$$

$$2.5071 G_y + 0.6445 = 0$$

$$2.5071 G_y = -0.6445$$

$$G_y = -0.2571 \text{ MPa}$$

$$G_x - 0.35 \cdot (-0.2571) + 0.1671 = 0$$

$$G_x = -0.2571 \text{ MPa}$$

$$E_z = \frac{1}{E} (G_z - \nu (G_x + G_y))$$

$$E_z = \frac{1}{477.5} (-0.4775 - 0.35 (-0.2571 - 0.2571))$$

$$E_z = -6.2310 \cdot 10^{-4}$$

7. ЗАДАЧА

$$P_z = 5 \text{ kN/m}^2 \quad A = a^2 \text{ [m}^2\text{]} \quad P_z = p \cdot A = 5a^2 \text{ kN}$$

1) E, ν - известны

$$\sigma_z = - \frac{P_z}{A} = - \frac{5a^2 \cdot 10^3}{a^2} = -5000 \text{ Pa}$$

$$\sigma_y = 0 \quad \sigma_y = \frac{1}{E} (G_y - \nu(G_x + G_z))$$

$$\sigma_x = 0 \text{ Pa} \quad 0 = \frac{1}{E} (G_x - \nu(G_y + 5000))$$

$$\tau_{xy} = 0 \quad 0 = G_{xy} - \nu(G_x - 5000)$$

$$\tau_{xz} = 0 \quad G_z = -\nu G_x - \nu 5000$$

$$\tau_{yz} = 0 \quad G_y = -\nu(G_x - 5000)$$

$$G_y = -\nu(0 - 5000) \quad \sigma_{xy} = 0$$

$$G_y = -\nu(-5000) \quad \sigma_{xz} = 0$$

$$G_y = 5000\nu \text{ Pa} \quad \sigma_{yz} = 0$$

$$\epsilon_x = \frac{1}{E} (G_x - \nu(G_y + G_z))$$

$$\epsilon_x = \frac{1}{E} (0 - \nu(5000\nu - 5000))$$

$$\epsilon_x = \frac{1}{E} (-5000\nu(\nu - 1)) = A \quad (*)$$

$$\epsilon_z = \frac{1}{E} (G_z - \nu(G_x + G_y))$$

$$\epsilon_z = \frac{1}{E} (-5000 - \nu(0 + 5000\nu))$$

$$\epsilon_z = \frac{1}{E} (-5000 - 5000\nu^2)$$

$$\epsilon_z = \frac{-5000}{E} (1 + \nu^2) = B \quad (*)$$

$$\frac{1}{E} (-5000\nu^2 + 5000\nu - 5000 - 5000\nu^2) =$$

$$= \frac{1}{E} (-3000\nu^2 + 5000\nu - 5000) = \frac{5000}{E} (-2\nu^2 + \nu - 1) =$$

$$= - \frac{5000}{E} (2\nu^2 - \nu + 1)$$

$$(-2\nu^2 + \nu - 1) : (\nu - 1) = -2\nu$$

$$\begin{array}{r} -2\nu^2 + 2\nu \\ + \quad - \\ \hline -\nu - 1 \end{array}$$

$$\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5000\nu & 0 \\ 0 & 0 & -5000 \end{bmatrix} \text{ Pa}$$

$$\sigma = \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{bmatrix}$$

$$\bar{\sigma} = 5000(\nu - 1)/3 =$$

$$= 1666.6667(\nu - 1)$$

$$\epsilon = - \frac{5000}{E} (2\nu^2 - \nu + 1)$$

$$\bar{\epsilon} = \frac{\epsilon}{3(1 - 2\nu)}$$

$$\epsilon = \frac{\bar{\epsilon}}{K}$$

$$K = \frac{\bar{\sigma}}{\epsilon}$$

$$K = \frac{5000(\nu - 1) \frac{E}{3(1 - 2\nu)}}{\frac{5000}{E} (-2\nu^2 + \nu - 1)} =$$

$$= \frac{E}{3(1 - 2\nu)}$$

$$5) \vec{n} = \begin{Bmatrix} 0 \\ 0.7071 \\ 0.7071 \end{Bmatrix} \quad \vec{m} = \begin{Bmatrix} 0 \\ 0.7071 \\ 0.7071 \end{Bmatrix}$$

$$g_{nm} = n^T D m = \begin{Bmatrix} 0 & 0 & 0.7071 & 0.7071 \end{Bmatrix} \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0.7071 \\ 0.7071 \end{Bmatrix} =$$

$$= \begin{Bmatrix} 0 & 0 & 0.7071 B \end{Bmatrix} \cdot \begin{Bmatrix} 0 \\ 0.7071 \\ 0.7071 \end{Bmatrix} = 0.7071^2 B = 0.5 B =$$

$$= \frac{1}{2} \cdot \left(\frac{-5000}{E} \right) (1 + \nu^2) =$$

$$= -\frac{2500}{E} (1 + \nu^2)$$

$$B) P_z = P = 5000 \text{ N/m}^2 \quad P_z = P_z \cdot A = 5000 \cdot 0.2 \text{ m}^2 \quad G_z = \frac{P_z}{A} = -\frac{5000 \text{ N}}{0.2 \text{ m}^2} = -25000 \text{ Pa}$$

$$P_x = P/2 = 2500 \text{ N/m}^2 \quad P_x = P_x \cdot A = -2500 \cdot 0.2 \text{ m}^2 \quad G_x = \frac{P_x}{A} = -\frac{2500 \text{ N}}{0.2 \text{ m}^2} = -12500 \text{ Pa}$$

$$A = 0.2 \text{ m}^2 \quad \epsilon_y = 0 \quad \epsilon_y = \frac{1}{E} (d_y - \nu (G_x + G_z))$$

$$E_y = 1103 \text{ N/m}^2$$

$$\tau_{xy} = 0 \quad g_{xy} = 0$$

$$\tau_{xz} = 0 \quad g_{xz} = 0$$

$$\tau_{yz} = 0 \quad g_{yz} = 0$$

$$S = \begin{bmatrix} -2500 & 0 & 0 \\ 0 & -7500 & 0 \\ 0 & 0 & -5000 \end{bmatrix}$$

$$D = \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{bmatrix}$$

$$\kappa = \frac{E}{3(1-2\nu)} \quad \bar{\kappa} = \frac{\bar{G}}{E}$$

$$\bar{G} = (-2500 - 7500 - 5000) = -15000$$

$$\bar{G} = -5000$$

$$C = A + B + 0 = \frac{2500}{E} (2\nu - 1 - 2 + \nu)$$

$$C = \frac{2500}{E} (3\nu - 3) = \frac{7500}{E} (\nu - 1)$$

$$0 = \frac{1}{E} (G_y - \nu (-5000 - 2500))$$

$$0 = G_y - \nu (-7500)$$

$$G_y = 7500 \nu$$

$$G_y = -7500 \nu$$

$$\epsilon_x = \frac{1}{E} (G_x + \nu (G_y + G_z))$$

$$\epsilon_x = \frac{1}{E} (-2500 - \nu (0 - 5000))$$

$$\epsilon_x = \frac{1}{E} (-2500 + 5000\nu)$$

$$\epsilon_x = \frac{2500}{E} (2\nu - 1) = A$$

$$\epsilon_z = \frac{1}{E} (G_z - \nu (G_x + G_y))$$

$$\epsilon_z = \frac{1}{E} (-5000 - \nu (-2500 + 0))$$

$$\epsilon_z = \frac{1}{E} (-5000 + 2500\nu)$$

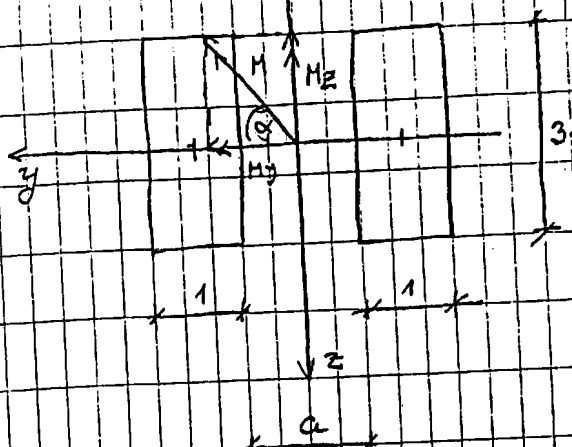
$$\epsilon_z = \frac{-2500}{E} (2 - \nu) = B$$

$$\frac{-5000E}{7500(\nu-1)} = \frac{E}{(1-\nu) \cdot 1.5} = \frac{E}{3(\frac{1}{2}-0.1\nu)}$$

ЧИСТО ПРАВО СВИЗАНЬЕ - ЧИСТО КОСО СВИЗАНЬЕ

$$\vec{G}_x = + \frac{M_z}{J_z} y + \frac{M_y}{J_y} z$$

1. ЗАДАЧА



$$M_z = M \sin \alpha$$

$$M_y = M \cos \alpha$$

$$J_z = 2 \cdot \left[\frac{1}{12} \cdot 1^3 \cdot 3 + \left(\frac{1}{2} + \frac{1}{2} \right)^2 \cdot 1 \cdot 3 \right]$$

$$J_z = 0.5 + 1.5(1+1)^2$$

$$J_y = 2 \cdot \frac{1}{12} \cdot 1 \cdot 3^3$$

$$J_y = 4.5$$

$$G_x = 0: \pi - \pi = \frac{M \sin \alpha}{0.5 + 1.5(1+1)^2} y + \frac{M \cos \alpha}{4.5} z = 0 \quad \left| \cdot \frac{4.5}{M} \right.$$

$$M - M = z = \tan(90^\circ - \alpha) y$$

$$z = -\tan \alpha$$

$$\frac{9 \sin \alpha}{1 + 3(1+1)^2} y + \frac{\cos \alpha}{1} z = 0$$

$$\cos \alpha z = - \frac{9 \sin \alpha}{1 + 3(1+1)^2} y$$

$$z = - \frac{9 \tan \alpha}{1 + 3(1+1)^2} y$$

$$- \frac{9 \tan \alpha}{1 + 3(1+1)^2} = - \tan \alpha \quad \left| \cdot \frac{1}{-\tan \alpha} \right.$$

$$\frac{9}{1 + 3(1+1)^2} = 1$$

$$1 + 3(1+1)^2 = \frac{9}{1}$$

$$1 + 3(1+1)^2 = 9$$

$$3(1+1)^2 = 9 - 1$$

$$3(1+1)^2 = 8$$

$$(1+1)^2 = 8/3$$

$$(1+1)^2 = 2.6667$$

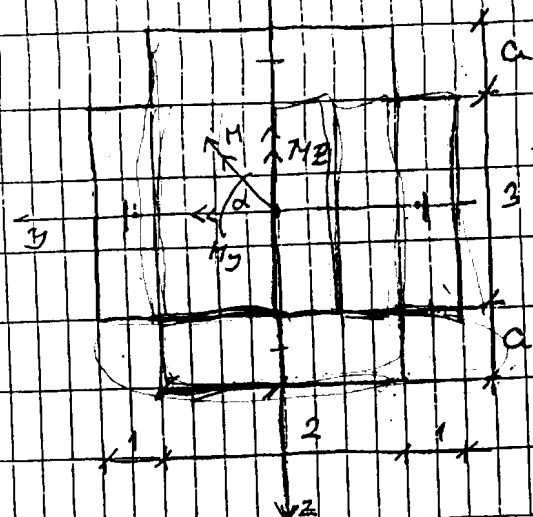
$$1+1 = \pm \sqrt{2.6667}$$

$$1+1 = \pm \sqrt{2.6667}$$

$$1+1 = 1.6330$$

$$1 = 1.6330 - 1$$

$$1 = 0.6330$$



$$Gx = \frac{F_z}{F_z} x + \frac{F_y}{F_y} y =$$

$$M_z = M \sin \alpha$$

$$M_y = M \cos \alpha$$

$$J_z = \frac{1}{12} \cdot (2a+3) \cdot 2^3 + 2 \left(\frac{1}{12} \cdot 3 \cdot 1^3 + 1.5^2 \cdot 1 \cdot 3 \right)$$

$$J_z = 0.6667 (2a+3) + 14$$

$$J_z = 1.3334 a + 16.0001$$

$$J_y = \frac{1}{12} \cdot 4 \cdot 3^3 + 2 \cdot \left(\frac{1}{12} \cdot 2 \cdot a^3 + \left(\frac{3}{2} + \frac{a}{2} \right)^2 \cdot 2 \cdot a \right)$$

$$J_y = 9 + 0.3333 a^3 + a(a+3)^2$$

$$Gx = 0 = n \cdot n = \frac{M \sin \alpha}{1.3334 a + 16.0001} y + \frac{M \cos \alpha}{9 + 0.3333 a^3 + a(a+3)^2} z = 0 \quad | \cdot \frac{1}{M}$$

$$\sin \alpha \cdot \frac{1}{1.3334 a + 16.0001} y = - \frac{\cos \alpha}{9 + 0.3333 a^3 + a(a+3)^2} z$$

$$\tan \alpha = \frac{a(a+3)^2 + 0.3333 a^3 + 9}{1.3334 a + 16.0001} y = z$$

$$M = M_z = z = \tan(360^\circ - \alpha) y$$

$$z = -\tan \alpha y$$

$$-\tan \alpha = \frac{a(a+3)^2 + 0.3333 a^3 + 9}{1.3334 a + 16.0001} = -\tan \alpha \quad | \cdot \frac{1}{\tan \alpha}$$

$$\frac{a(a+3)^2 + 0.3333 a^3 + 9}{1.3334 a + 16.0001} = 1$$

$$a(a^2 + 6a + 9) + 0.3333 a^3 + 9 = 1.3334 a + 16.0001$$

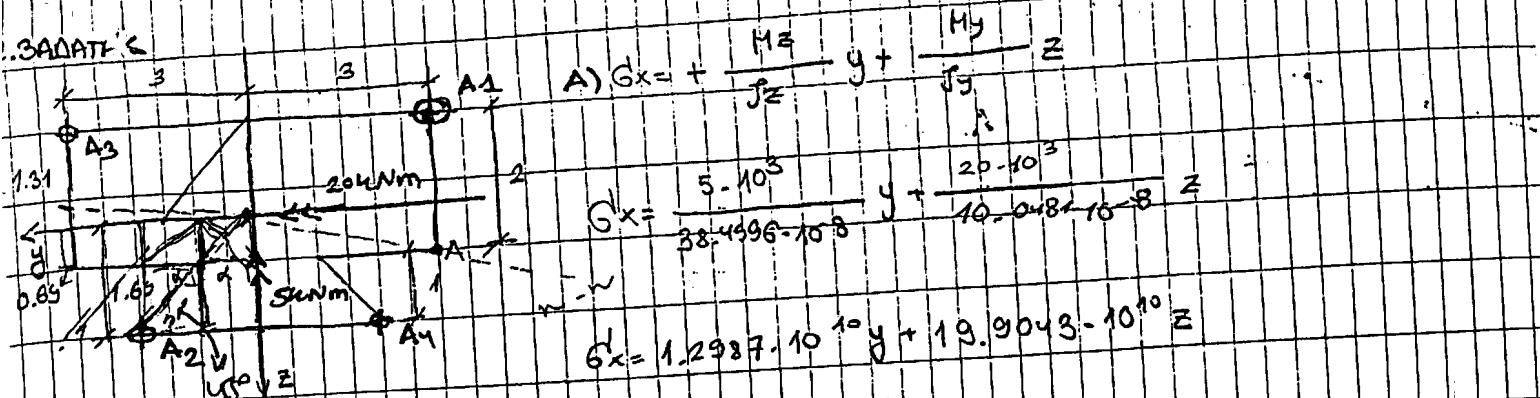
$$1.3333 a^3 + 6a^2 + 7.6666 a - 7.0001 = 0$$

$$a_1 = \} \in \mathbb{C}$$

$$a_2 = \}$$

$$a_3 = 0.5971$$

3. ЗАДАЧА



$J_y = 10.0481 \text{ cm}^4$ $M_y = 20 \text{ kNm}$

$J_z = 38.4996 \text{ cm}^4$ $M_z = 5 \text{ kNm}$

B) $W_y = \frac{J_y}{z_{\max}} = \frac{10.0481}{\pm 1.63} = 5.9456 \text{ cm}^3$

$W_z = \frac{J_z}{y_{\max}} = \frac{38.4996}{\pm 3} = 12.8332 \text{ cm}^3$

B) $G_x = 0 : n-n : z = \frac{1.2987 \cdot 10^{10}}{-19.9043 \cdot 10^{10}} y \quad z = -0.0652 y$

Г) $G_{x,A} = 1.2987 \cdot 10^{10} \cdot (-3) \cdot 10^{-2} + 19.9043 \cdot 10^{10} \cdot 0.69 \cdot 10^{-2}$

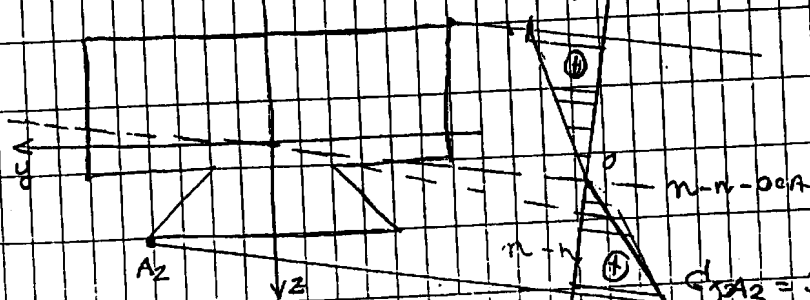
$A(-3; +0.69) \quad G_{x,A} = 983.7867 \text{ MPa}$

Δ) $\tau_{\max} = \frac{1}{2} |G_{\max}| = \frac{1}{2} \cdot 3623.5667 = 1811.7834 \text{ MPa}$

⊕ A1 (-3; -1.31) $G_{x,A1} = 1.2987 \cdot 10^{10} \cdot (-3) \cdot 10^{-2} + 19.9043 \cdot 10^{10} \cdot (-1.31) \cdot 10^{-2} = -2997.0733 \text{ MPa}$

⊖ A2 (2; 1.63) $G_{x,A2} = 1.2987 \cdot 10^{10} \cdot 2 \cdot 10^{-2} + 19.9043 \cdot 10^{10} \cdot 1.63 \cdot 10^{-2} = 3623.5667 \text{ MPa}$

A1 $G_{x,A1} = -2997.0733$



$M_y = 15 \text{ kNm} \quad J_y = 12.0000 \text{ cm}^4$
 $M_z = 20 \text{ kNm} \quad J_z = 46.8331 \text{ cm}^4$

$A) G_x = -\frac{M_z}{J_z} y + \frac{M_y}{J_y} z$

$G_x = -\frac{20 \cdot 10^3}{46.8331 \cdot 10^{-8}} y + \frac{15 \cdot 10^3}{12.0000 \cdot 10^{-8}} z$

$G_x = -4.2705 \cdot 10^{10} y + 12.5 \cdot 10^{10} z$

$b) W_y = \frac{J_y}{z_{max}} = \frac{12.0000}{1.56} = 7.6923 \text{ cm}^3$

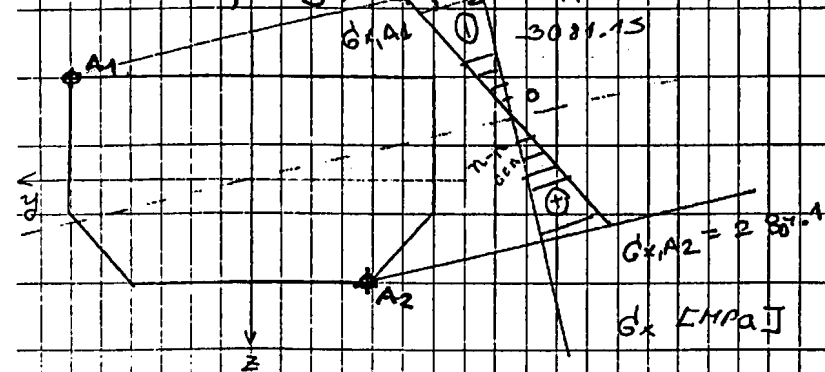
$W_z = \frac{J_z}{z_{max}} = \frac{46.8331}{3} = 15.6110 = 15.6110 \text{ cm}^3$

$b) n-n: G_x = 0 \quad z = \frac{-4.2705 \cdot 10^{10}}{12.5 \cdot 10^{10}} y \quad z = 0.3416 y$

$\Gamma) A(-3; -1.44) \quad G_{x,A} = -4.2705 \cdot 10^{10} \cdot (-3) \cdot 10^{-2} + 12.5 \cdot 10^{10} \cdot (-1.44) \cdot 10^{-2}$
 $G_{x,A} = -518.85 \text{ MPa}$

$\Delta) A_1(3; -1.44) \quad G_{x,A1} = -4.2705 \cdot 10^{10} \cdot 3 \cdot 10^{-2} + 12.5 \cdot 10^{10} \cdot (-1.44) \cdot 10^{-2} = -3081.15 \text{ MPa}$

$A_2(-2; 1.56) \quad G_{x,A2} = -4.2705 \cdot 10^{10} \cdot (-2) \cdot 10^{-2} + 12.5 \cdot 10^{10} \cdot 1.56 \cdot 10^{-2} = 2804.1 \text{ MPa}$



3. ЗАДАЧА

[cm]

$$G_x = -\frac{M_z}{J_z} y + \frac{M_y}{J_y} z$$

$$M_z = M \cdot \cos \alpha = 3 \cdot 10^3 \cdot \cos 45^\circ = 2.1213 \text{ kNm}$$

$$M_y = M \cdot \sin \alpha = 3 \cdot 10^3 \cdot \sin 45^\circ = 2.1213 \text{ kNm}$$

$$J_z = \frac{1}{12} \cdot (2\sqrt{2})^4 + 2 \cdot \left(\frac{1}{48} (2\sqrt{2})^3 \cdot \sqrt{2} \right)$$

$$J_z = 6.6667 \text{ cm}^4$$

$$J_y = \frac{1}{12} \cdot (2\sqrt{2})^4 + 2 \cdot \left(\frac{1}{48} (2\sqrt{2})^3 \cdot \sqrt{2} \right)$$

$$J_y = 20 \text{ cm}^4$$

$$G_x = -\frac{2.1213 \cdot 10^3}{6.6667 \cdot 10^{-8}} y + \frac{2.1213 \cdot 10^3}{20 \cdot 10^{-8}} z$$

$$G_x = -3.1819 \cdot 10^{10} y + 1.0606 \cdot 10^{10} z$$

$$G_x = 0 \Rightarrow n-n: z = \frac{-3.1819 \cdot 10^{10}}{1.0606 \cdot 10^{10}} y$$

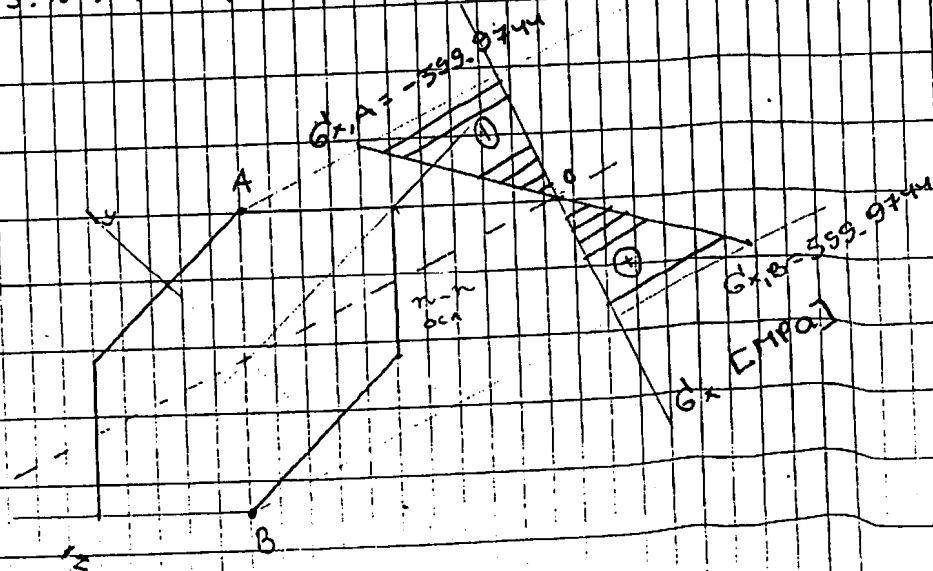
$$z = 3.0001 y$$

$$A'(0, -2) : \begin{cases} y_A = y \cos 315^\circ + z \sin 315^\circ = 0 \cdot 0.7071 + 2 \cdot 0.7071 = +1.4142 \text{ cm} \\ z_A = -y \sin 315^\circ + z \cos 315^\circ = +0 \cdot 0.7071 - 2 \cdot 0.7071 = -1.4142 \text{ cm} \end{cases}$$

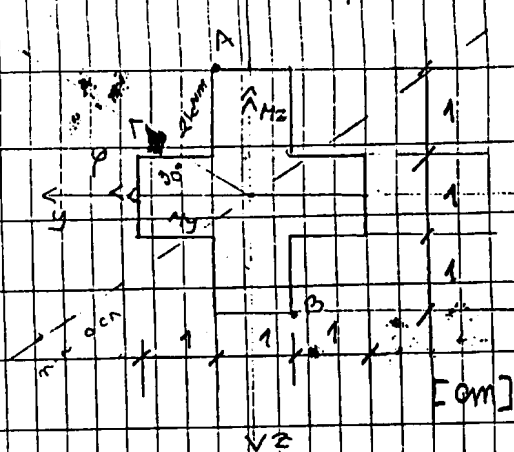
$$G_{x,A} = -3.1819 \cdot 10^{10} \cdot 1.4142 \cdot 10^{-2} + 1.0606 \cdot 10^{10} \cdot (-1.4142) \cdot 10^{-2} = -559.9744 \text{ MPa}$$

$$B'(0, 2) : \begin{cases} y_B = y \cos 315^\circ + z \sin 315^\circ = 0 \cdot 0.7071 + 2 \cdot 0.7071 = +1.4142 \text{ cm} \\ z_B = -y \sin 315^\circ + z \cos 315^\circ = +0 \cdot 0.7071 + 2 \cdot 0.7071 = +1.4142 \text{ cm} \end{cases}$$

$$G_{x,B} = -3.1819 \cdot 10^{10} \cdot (-1.4142 \cdot 10^{-2}) + 1.0606 \cdot 10^{10} \cdot 1.4142 \cdot 10^{-2} = 559.9744 \text{ MPa}$$



$$G_x = + \frac{M_z}{J_z} y + \frac{M_y}{J_y} z$$



$$M_y = M \cos 30^\circ = 2 \cdot 10^3 \cdot \cos 30^\circ = 1.7320 \text{ kNm}$$

$$M_z = M \sin 30^\circ = 2 \cdot 10^3 \cdot \sin 30^\circ = 1 \text{ kNm}$$

$$J_y = \frac{1}{12} \cdot 3 \cdot 1^3 + 2 \cdot \left(\frac{1}{12} \cdot 1^4 + 1^2 \cdot 1^2 \right)$$

$$J_y = 2.4167 \text{ cm}^4$$

$$J_z = \frac{1}{12} \cdot 3 \cdot 1^3 + 2 \cdot \left(\frac{1}{12} \cdot 1^4 + 1^2 \cdot 1^2 \right) = J_z = 2.4167 \text{ cm}^4$$

$$G_x = \frac{10^3}{2.4167 \cdot 10^{-8}} y + \frac{1.7320 \cdot 10^3}{2.4167 \cdot 10^{-8}} z$$

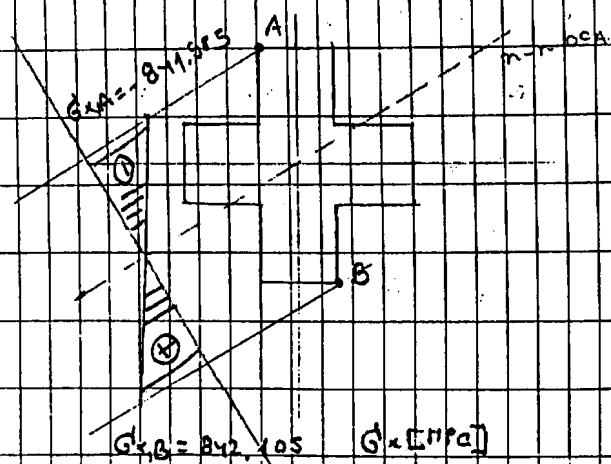
$$G_x = 4.6583 \cdot 10^{10} y + 7.1668 \cdot 10^{10} z \quad G_x = 0 \quad n \perp oca:$$

$$n \perp n: \quad z = \frac{4.6583 \cdot 10^{10}}{-7.1668 \cdot 10^{10}} y$$

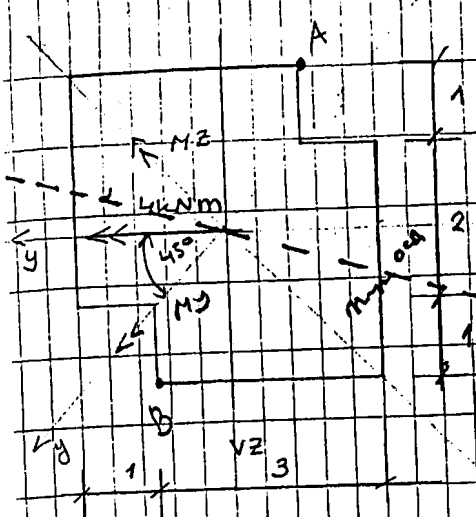
$$z = -0.65 y$$

$$A(0.5; -1.5) \quad G_{x,A} = 4.6583 \cdot 10^{10} \cdot 0.5 \cdot 10^{-2} + 7.1668 \cdot 10^{10} \cdot (-1.5) \cdot 10^{-2} = -811.985 \text{ MPa}$$

$$B(-0.5; 1.5) \quad G_{x,B} = 4.6583 \cdot 10^{10} \cdot (-0.5) \cdot 10^{-2} + 7.1668 \cdot 10^{10} \cdot (1.5) \cdot 10^{-2} = 842.105 \text{ MPa}$$



$$\sigma_x = + \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$



$$I_{y1} = \frac{1}{12} \cdot 4^4 - 2 \left(\frac{1}{12} \cdot 1^4 + 1.5^2 \cdot 1^2 \right) = 16.6667 \text{ cm}^4$$

$$I_{z1} = \frac{1}{12} \cdot 4^4 - 2 \left(\frac{1}{12} \cdot 1^4 + 1.5^2 \cdot 1^2 \right) = 16.6667 \text{ cm}^4$$

$$I_{yz1} = 0 - 2 \cdot (0 + 1.5^2 \cdot 1^2) = -5 \text{ cm}^4$$

$$I_{y12} = \frac{16.667 \cdot 2}{2} \pm \sqrt{(-4.5)^2} = I_{y1} = 21.1667 \text{ cm}^4$$

$$I_{z12} = 12.1667 \text{ cm}^4$$

$$\tan 2\alpha = \frac{-2 \cdot (-4.5)}{16.6667 - 16.6667} = \frac{9}{0} = \infty$$

$$M_z = M \sin \varphi = 4 \cdot 10^3 \cdot \sin 45^\circ$$

$$\alpha_1 = 45^\circ$$

$$M_z = 2.8284 \text{ kNm}$$

$$\alpha_2 = 135^\circ$$

$$M_y = M \cos \varphi = 4 \cdot 10^3 \cdot \cos 45^\circ$$

$$\sigma_x = \frac{2.8284 \cdot 10^3}{12.1667 \cdot 10^{-8}} y + \frac{2.8284 \cdot 10^3}{21.1667 \cdot 10^{-8}} z$$

$$M_y = 2.8284 \text{ kNm}$$

$$I_{y1} = 21.1667 \text{ cm}^4$$

$$\sigma_x = 2.3247 \cdot 10^{10} y + 1.3362 \cdot 10^{10} z$$

$$I_{z1} = 12.1667 \text{ cm}^4$$

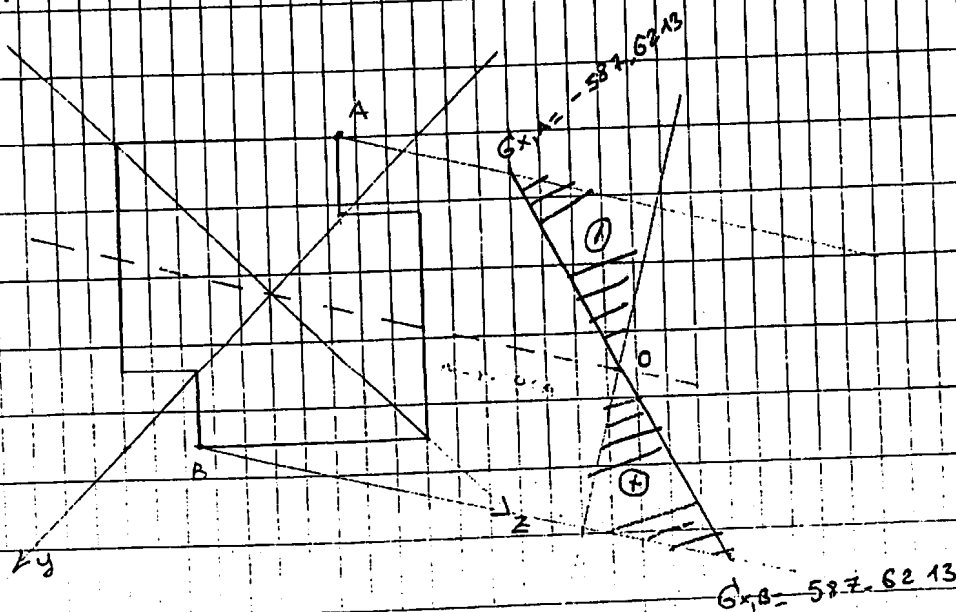
$$\sigma_x = 0: z = \frac{2.3247 \cdot 10^{10}}{-1.3362 \cdot 10^{10}} y$$

$$z = -1.7398 y$$

$$\begin{aligned} A(-1; -2) &= \begin{cases} y_A = y \cos \alpha_1 + z \sin \alpha_1 = -1 \cdot \cos 45^\circ + 2 \cdot \sin 45^\circ = -2.1213 \text{ cm} \\ z_A = -y \sin \alpha_1 + z \cos \alpha_1 = 1 \sin 45^\circ + 2 \sin 45^\circ = -0.7071 \text{ cm} \end{cases} \\ B(1; 2) &= \begin{cases} y_B = y \cos \alpha_1 + z \sin \alpha_1 = 1 \cos 45^\circ + 2 \sin 45^\circ = 2.1213 \text{ cm} \\ z_B = -y \sin \alpha_1 + z \cos \alpha_1 = -1 \sin 45^\circ + 2 \cos 45^\circ = 0.7071 \text{ cm} \end{cases} \end{aligned}$$

$$\sigma_{x,A} = 2.3247 \cdot 10^{10} \cdot (-2.1213 \cdot 10^{-2}) + 1.3362 \cdot 10^{10} \cdot (-0.7071 \cdot 10^{-2}) = -587.6213 \text{ MPa}$$

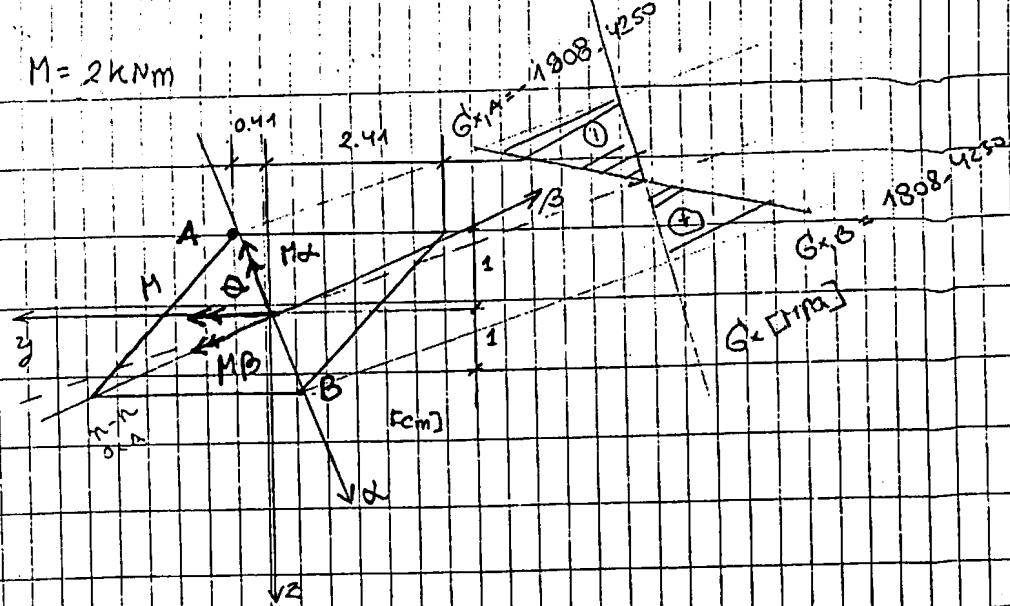
$$\sigma_{x,B} = 2.3247 \cdot 10^{10} \cdot 2.1213 \cdot 10^{-2} + 1.3362 \cdot 10^{10} \cdot 0.7071 \cdot 10^{-2} = 587.6213 \text{ MPa}$$



σ_x [MPa]



$$M = 2 \text{ kNm}$$



$$J_y = 1.8856$$

$$J_{12} = \frac{1.8856 + 5.6569}{2} = \sqrt{\left(\frac{1.8856 - 5.6569}{2}\right)^2 + 1.3356^2}$$

$$J_z = 5.6569$$

$$J_y = 1.8856$$

$$J_1 = 6.4379 \text{ cm}^4 = J_{\alpha} \text{ to } 2 \alpha_{112} = \frac{-2 \cdot 1.8856}{1.8856 - 5.6569}$$

$$\alpha_1 = 112.500^\circ$$

$$J_2 = 1.1046 \text{ cm}^4 = J_{\beta}$$

$$\alpha_2 = 202.500^\circ$$

$$\theta = 180^\circ - \alpha_1 = 180^\circ - 112.500^\circ = 67.5^\circ$$

$$G_x = - \frac{M \alpha}{J_{\alpha}} \beta + \frac{M \beta}{J_{\beta}} \alpha$$

$$M_{\alpha} = M \cdot \cos \theta = 2 \cdot \cos 67.5^\circ = 0.7634 \text{ kNm}$$

$$M_{\beta} = M \sin \theta = 2 \cdot \sin 67.5^\circ = 1.8473 \text{ kNm}$$

$$G_x = \frac{1.8473 \cdot 10^3}{1.1046 \cdot 10^{-8}} \alpha + \frac{0.7634 \cdot 10^3}{6.4379 \cdot 10^{-8}} \beta$$

$$G_x = 1.6728 \cdot 10^{11} \alpha - 1.1889 \cdot 10^{10} \beta \quad G_x = 0 : 1 + n = \beta = \frac{1.6728 \cdot 10^{11}}{1.1889 \cdot 10^{10}} \alpha$$

$$\beta = 14.0701 \alpha$$

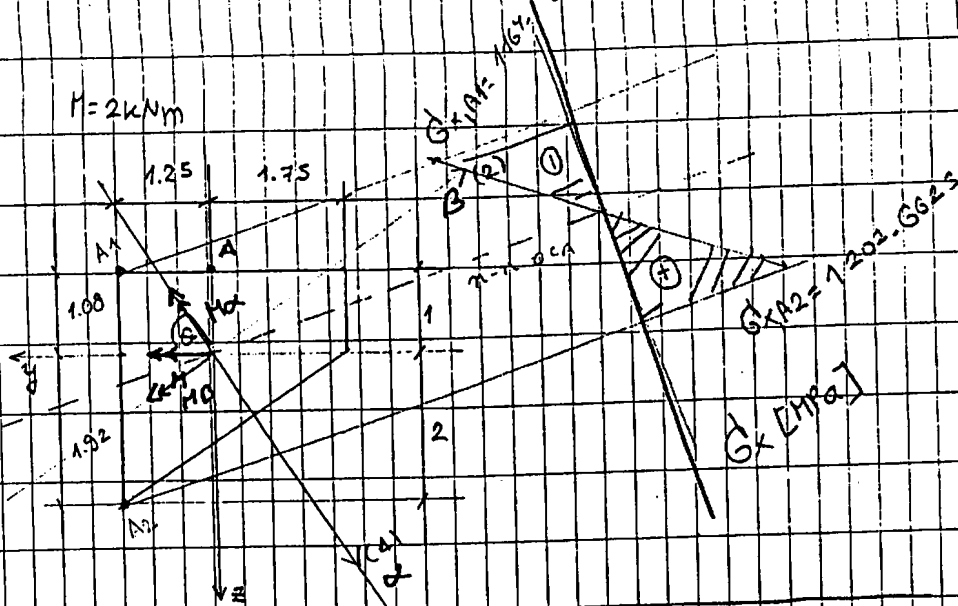
$$A(0.41; -1) = \begin{cases} \alpha_A = 0.41 \cos 112.5^\circ - \sin 112.5^\circ = -1.0808 \text{ cm} \\ \beta_A = -0.41 \sin 112.5^\circ - \cos 112.5^\circ = 3.8928 \cdot 10^{-3} \text{ cm} \end{cases}$$

$$G_{x,A} = 1.6728 \cdot 10^{11} \cdot (-1.0808) \cdot 10^{-2} - 1.1889 \cdot 10^{10} \cdot 3.8928 \cdot 10^{-5} = -1808.4250 \text{ MPa}$$

$$B(-0.41; 1) = \begin{cases} \alpha_B = -0.41 \cos 112.5^\circ + \sin 112.5^\circ = 1.0808 \text{ cm} \\ \beta_B = 0.41 \sin 112.5^\circ + \cos 112.5^\circ = -3.8928 \cdot 10^{-3} \text{ cm} \end{cases}$$

$$G_{x,B} = 1.6728 \cdot 10^{11} \cdot (1.0808) \cdot 10^{-2} - 1.1889 \cdot 10^{10} \cdot (-3.8928) \cdot 10^{-5} = 1808.4250 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2} |G_{x,\max}| = \frac{1}{2} 1808.4250 = 904.2125 \text{ MPa}$$



$$J_y = 2.9583 \quad J_{y12} = \frac{2.9583 + 4.1250}{2} \pm \sqrt{\left(\frac{2.9583 - 4.1250}{2}\right)^2 + 1.3750^2} \quad \tan 2\alpha_{1/2} = \frac{-2.13750}{1.2.9583 - 4.1250}$$

$$J_z = 4.1250$$

$$J_{yz} = 1.3750 \quad J_1 = 5.0353 \text{ cm}^4 \quad \alpha_1 = 123.5053^\circ$$

$$J_2 = 2.0480 \text{ cm}^4 \quad \alpha_2 = 213.5053^\circ$$

$$\Theta = 180^\circ - \alpha_1 = 180^\circ - 123.5053^\circ = 56.4947^\circ$$

$$G_x = + \frac{M_B}{J_B} \alpha - \frac{M_A}{J_A} \beta$$

$$M_B = M \cdot \sin \Theta = 2 \cdot 10^3 \cdot \sin 56.4947^\circ = 1.6677 \text{ kNm}$$

$$M_A = M \cdot \cos \Theta = 2 \cdot 10^3 \cdot \cos 56.4947^\circ = 1.1040 \text{ kNm}$$

$$G_x = \frac{1.6677 \cdot 10^3}{2.0480 \cdot 10^{-4}} \alpha - \frac{1.1040 \cdot 10^3}{5.0353 \cdot 10^{-4}} \beta \quad \text{in the OCA, } G_x = 0: \quad \beta = \frac{8.1431 \cdot 10^{10}}{2.1925 \cdot 10^{10}} \alpha$$

$$G_x = 8.1431 \cdot 10^{10} \alpha - 2.1925 \cdot 10^{10} \beta \quad \beta = 3.7141 \alpha$$

$$A(0,1) = \begin{cases} \alpha_A = 0 \cdot \cos 123.5053^\circ - \sin 123.5053^\circ = -0.8338 \text{ cm} \\ \beta_A = 0 \cdot \sin 123.5053^\circ - \cos 123.5053^\circ = 0.5520 \text{ cm} \end{cases}$$

$$G_{xA} = 8.1431 \cdot 10^{10} \cdot (-0.8338) \cdot 10^{-2} - 2.1925 \cdot 10^{10} \cdot 0.5520 \cdot 10^{-1} = -743.9766 \text{ MPa}$$

$$A1(1.25; -1.08) = \begin{cases} \alpha_{A1} = 1.25 \cos 123.5053^\circ - 1.08 \sin 123.5053^\circ = -1.5906 \text{ cm} \\ \beta_{A1} = -1.25 \sin 123.5053^\circ - 1.08 \cos 123.5053^\circ = -0.4461 \text{ cm} \end{cases}$$

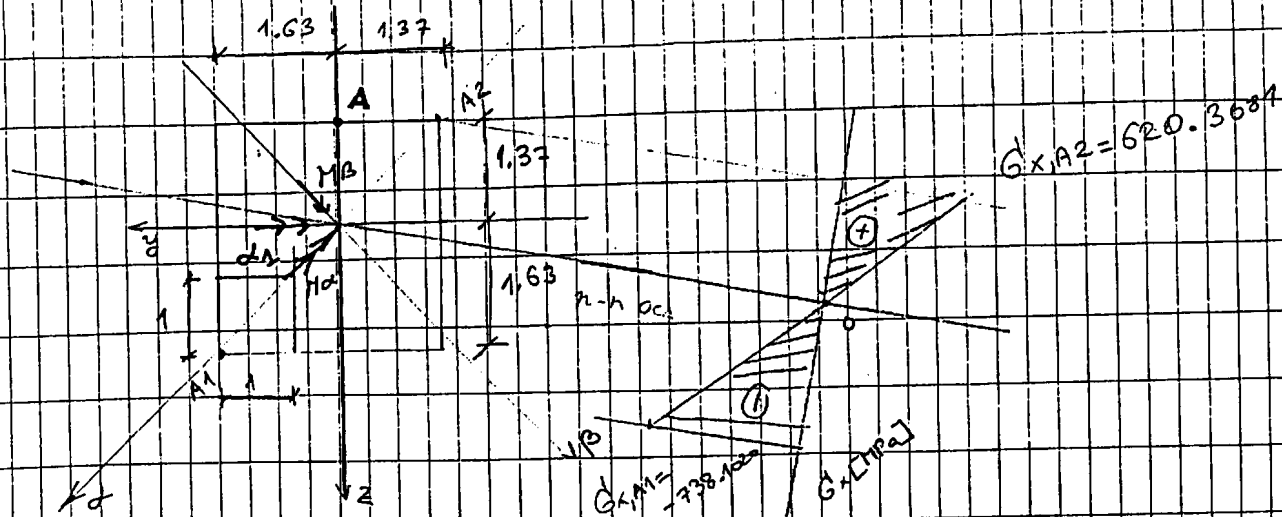
$$G_{xA1} = 8.1431 \cdot 10^{10} \cdot (-1.5906) \cdot 10^{-2} - 2.1925 \cdot 10^{10} \cdot (-0.4461) \cdot 10^{-1} = -1.164.8617 \text{ MPa}$$

$$A2(1.25; 1.92) = \begin{cases} \alpha_{A2} = 1.25 \cos 123.5053^\circ + 1.92 \sin 123.5053^\circ = 0.9109 \text{ cm} \\ \beta_{A2} = -1.25 \sin 123.5053^\circ + 1.92 \cos 123.5053^\circ = -2.1022 \text{ cm} \end{cases}$$

$$G_{xA2} = 8.1431 \cdot 10^{10} \cdot 0.9109 \cdot 10^{-2} - 2.1925 \cdot 10^{10} \cdot (-2.1022) \cdot 10^{-1} = 1.202.6623 \text{ MPa}$$

$$\tau_{m_{xz}} = \frac{1}{2} |G_{x, m_{xz}}| = \frac{1}{2} 1202.6623 = 601.3312 \text{ MPa}$$

$$M = 2 \text{ kNm}$$



$$J_y = 5.5417 \quad J_{yz} = 5.5417 \pm \sqrt{(-1.1250)^2} \quad \tan 2\alpha_{1/2} = \frac{-2 \cdot (-1.1250)}{5.5417 - 5.5417}$$

$$J_z = 5.5417 \quad J_1 = J_{\alpha} = 6.6667 \text{ cm}^4 \quad \alpha_1 = 45^\circ$$

$$J_{yz} = -1.1250 \quad J_2 = J_{\beta} = 4.4167 \text{ cm}^4 \quad \alpha_2 = 135^\circ$$

$$G_x = -\frac{M_{\beta}}{J_{\beta}} \alpha - \frac{M_{\alpha}}{J_{\alpha}} \beta \quad M_{\beta} = M \cdot \cos \alpha_1 = 2 \cdot 10^3 \cdot \cos 45^\circ = 1.4142 \text{ kNm}$$

$$M_{\alpha} = M \cdot \sin \alpha_1 = 2 \cdot 10^3 \cdot \sin 45^\circ = 1.4142 \text{ kNm}$$

$$G_x = -\frac{1.4142 \cdot 10^3}{4.4167 \cdot 10^{-8}} \alpha - \frac{1.4142 \cdot 10^3}{6.6667 \cdot 10^{-8}} \beta \quad G_x = 0 \quad \beta = \frac{-3.2019 \cdot 10^{10}}{2.1213 \cdot 10^{10}} \alpha$$

$$G_x = -3.2019 \cdot 10^{10} \alpha - 2.1213 \cdot 10^{10} \beta \quad \beta = -1.5094 \alpha$$

$$A(0; -1.37) \quad \begin{cases} \alpha_A = 0 \cdot \cos 45^\circ - 1.37 \cdot \sin 45^\circ = -0.9687 \text{ cm} \\ \beta_A = -0 \cdot \sin 45^\circ - 1.37 \cdot \cos 45^\circ = -0.9687 \text{ cm} \end{cases}$$

$$\beta_A = -0 \cdot \sin 45^\circ - 1.37 \cdot \cos 45^\circ = -0.9687 \text{ cm}$$

$$G_x = -3.2019 \cdot 10^{10} \cdot (-0.9687) \cdot 10^{-2} - 2.1213 \cdot 10^{10} \cdot (-0.9687) \cdot 10^{-4} = 515.6584 \text{ MPa}$$

$$A(1.63; 1.63) \quad \begin{cases} \alpha_{A1} = 1.63 \cos 45^\circ + 1.63 \sin 45^\circ = 2.3052 \text{ cm} \\ \beta_{A1} = -1.63 \sin 45^\circ + 1.63 \cos 45^\circ = 0 \end{cases}$$

$$\beta_{A1} = -1.63 \sin 45^\circ + 1.63 \cos 45^\circ = 0$$

$$G_{xA1} = -3.2019 \cdot 10^{10} \cdot 2.3052 \cdot 10^{-2} = -738.1020 \text{ MPa}$$

$$A2(-1.37, -1.37) \quad \begin{cases} \alpha_{A2} = -1.37 \cos 45^\circ + 1.37 \sin 45^\circ = -1.9375 \text{ cm} \\ \beta_{A2} = 1.37 \sin 45^\circ - 1.37 \cos 45^\circ = 0 \end{cases}$$

$$\beta_{A2} = 1.37 \sin 45^\circ - 1.37 \cos 45^\circ = 0$$

$$G_{xA2} = -3.2019 \cdot 10^{10} \cdot (-1.9375) \cdot 10^{-2} = 620.3681 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2} |G_{x, \max}| = \frac{1}{2} \cdot 738.1020 \text{ MPa} = 369.051 \text{ MPa}$$

~ АКСИЈАЛНО НАПРЕЗАЊЕ ГРЕДНОГ НОСАЧА ~

А) ФУНКЦИЈА ОПТЕРЕБЕЊА ШТАПА

δ -силеауфична штеина маиеруана су коа де нана рабрен штеина

$$Q_x(x) = \delta V_x(x)$$

$$A(x) = a(x) \cdot b$$

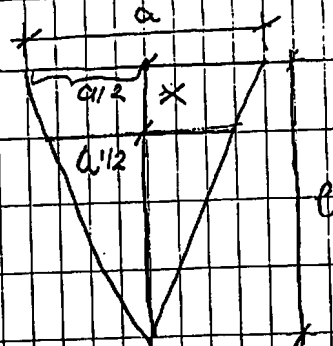
$$Q_x(x) = \delta \cdot A(x) \cdot dx$$

$$A(x) = A_0 \left(\frac{e-x}{e} \right) \cdot b$$

$$Q_x(x) = \delta \cdot A_0 \cdot \left(\frac{e-x}{e} \right) dx$$

$$A_0 = a \cdot b$$

$$Q_x(x) = \delta A_0 \cdot \left(\frac{e-x}{e} \right) dx$$



$$a/2 \cdot e = a'/2 \cdot (e-x)$$

$$e \cdot \frac{a'}{2} = \frac{a}{2} \cdot (e-x)$$

$$e \cdot a' = a(e-x)$$

$$a' = a \left(\frac{e-x}{e} \right)$$

$$b) u(0) = 0$$

$$N(e) = 0$$

$$c) \frac{dN}{dx} = -Q_x(x)$$

$$dN = -Q_x(x) dx$$

$$\int dN = - \int Q_x(x) dx + C_1$$

$$N(x) = - \int \delta A_0 \left(\frac{e-x}{e} \right) dx + C_1 \quad \text{смена } \frac{e-x}{e} = t$$

$$N(x) = - \int \delta A_0 \cdot t \cdot (-e) dt + C_1 \quad - \frac{dx}{e} = dt$$

$$N(x) = \delta A_0 e \int t dt + C_1 \quad dx = -e dt$$

$$N(x) = \delta A_0 e \frac{t^2}{2} + C_1$$

$$N(x) = \delta A_0 \frac{e}{2} \cdot \left(\frac{e-x}{e} \right)^2 + C_1$$

$$N(e) = 0 \quad N(e) = \delta A_0 \frac{e}{2} \left(\frac{e-e}{e} \right)^2 + C_1$$

$$N(e) = C_1 \quad N(e) = 0 + C_1$$

$$C_1 = 0 \quad N(e) = C_1$$

$$N(x) = \delta A_0 \frac{e}{2} \left(\frac{e-x}{e} \right)^2$$

$$u(x) = \int \epsilon_x(x) dx + C_2$$

$$\epsilon_x(x) = \frac{N(x)}{A(x)} = \frac{\delta A_0 \frac{e}{2} \left(\frac{e-x}{e} \right)^2}{A_0 \left(\frac{e-x}{e} \right)} = \delta \frac{e}{2} \left(\frac{e-x}{e} \right)$$

$$\epsilon_x(x) = \delta \frac{e}{2} \left(\frac{e-x}{e} \right)$$

$$\epsilon_x(x) = \frac{G_x(x)}{E} = \frac{\delta e \left(\frac{e-x}{e} \right)}{2E}$$

$$\epsilon_x(x) = \frac{\delta e}{2E} \left(\frac{e-x}{e} \right)$$

$$u(x) = \int \frac{\delta e}{2E} \left(\frac{e-x}{e} \right) dx + C_2$$

$$u(x) = \int \frac{\delta e}{2E} (e-x) dx + C_2$$

$$u(x) = \frac{\delta e}{2E} \left(ex - \frac{x^2}{2} \right) + C_2$$

$$u(0) = 0$$

$$u(0) = \frac{\delta}{2E} (0 - 0) + C_2$$

$$u(0) = C_2$$

$$C_2 = 0$$

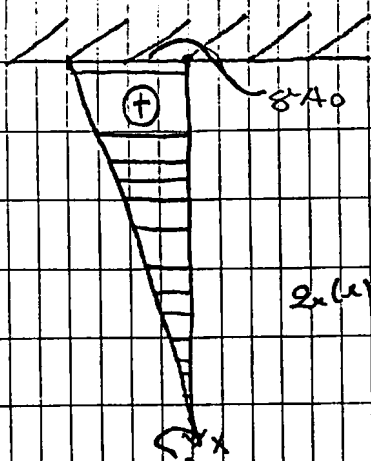
$$u(x) = \frac{\delta e}{2E} \left(ex - \frac{x^2}{2} \right)$$

31/10/17

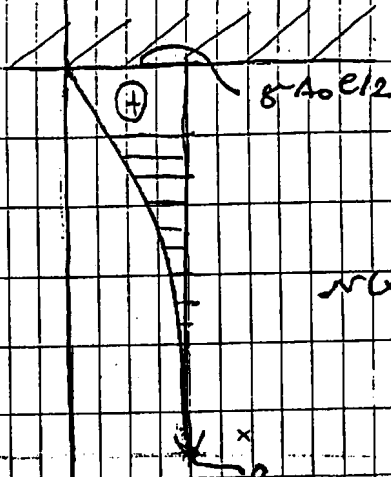
ДИГРАМЫ

$$q_x(x) = 8A_0 \left(\frac{e-x}{e} \right) \quad x=0 \quad q_x(0) = 8A_0 \quad x=e \quad q_x(e) = 0$$

$$N(x) = 8A_0 \frac{e}{2} \left(\frac{e-x}{e} \right)^2 \quad x=0 \quad N(0) = 8A_0 \frac{e}{2} \quad x=e \quad N(e) = 0$$



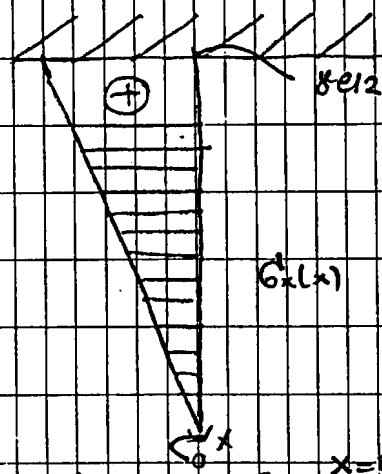
$q_x(x)$



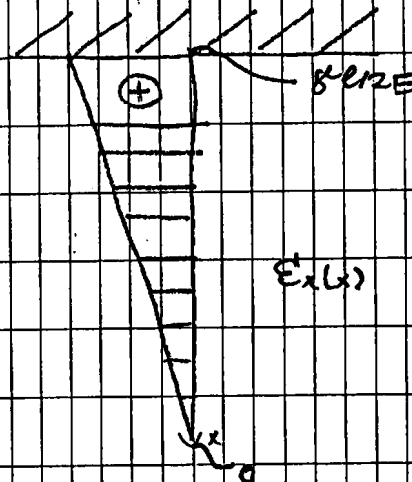
$N(x)$

$$G_x(x) = 8 \frac{e}{2} \left(\frac{e-x}{e} \right) \quad x=0 \quad G_x(0) = 8e/2 \quad x=e \quad G_x(e) = 0$$

$$E_x(x) = \frac{8e}{2E} \left(\frac{e-x}{e} \right) \quad x=0 \quad E_x(0) = 8e/2E \quad x=e \quad E_x(e) = 0$$

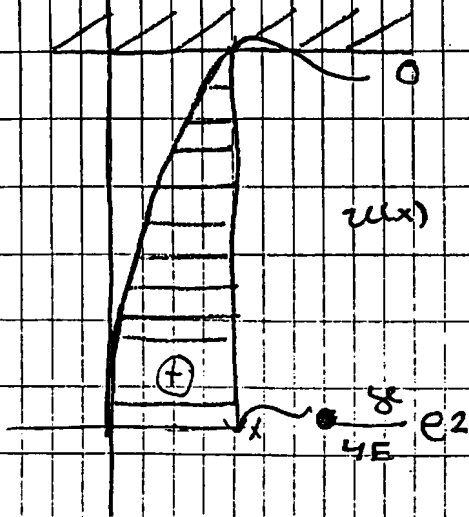


$G_x(x)$



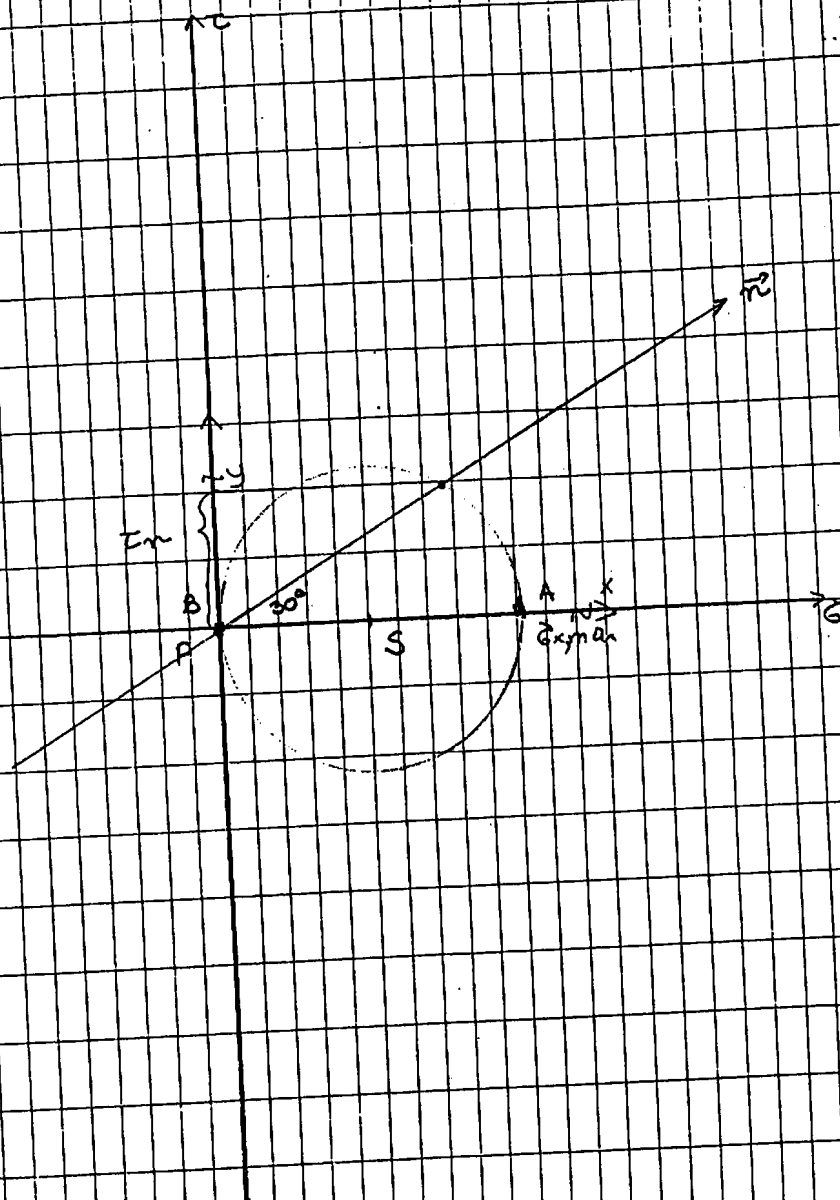
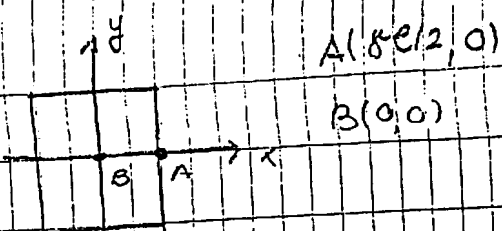
$E_x(x)$

$$U(x) = \frac{8}{2E} \left(ex - \frac{x^2}{2} \right) \quad x=0 \quad U(0) = 0 \quad x=e \quad U(e) = \frac{8}{4E} e^2$$



$U(x)$

$$S = \begin{bmatrix} \frac{8e}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$$\frac{8e}{2} = G_{x\max} = 40m$$

$$r_{200} = r_n = 1.8cm = 0.2258e$$

$$\frac{8e}{2} : 4 = x = 1.8$$

$$4x = 1.8 \cdot \frac{8e}{2}$$

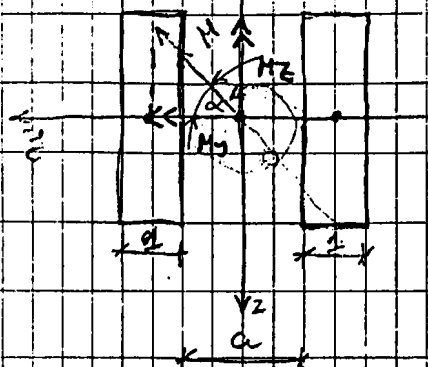
$$4x = 0.98e$$

$$x = \frac{0.9}{4} 8e$$

$$x = 0.2258e$$

ЧИСТО ПРАВО СЪВИЗАЮЩЕ - ЧИСТО КОСО СЪВИЗАЮЩЕ

1. ЗАДАЧА



$$G_x = + \frac{M_z}{J_z} \bar{z} + \frac{M_y}{J_y} \bar{y}$$

$$M_z = M \sin \alpha$$

$$M_y = M \cos \alpha$$

$$J_z = \left[\frac{1}{12} \cdot 1^3 \cdot 3 + \left(\frac{0}{2} + \frac{1}{2} \right)^2 \cdot 1 \cdot 3 \right] \cdot 2$$

$$J_z = [0.25 + 0.75(a+1)^2] \cdot 2$$

$$J_z = 0.5 + 1.5(a+1)^2$$

$$J_y = \left[\frac{1}{12} \cdot 1 \cdot 3^3 \right] \cdot 2$$

$$J_y = 4.5$$

$$G_x = 0 \quad \frac{M \sin \alpha}{0.5 + 1.5(a+1)^2} \cdot y + \frac{M \cos \alpha}{4.5} \cdot z = 0 \quad | \cdot \frac{1}{M}$$

$$z = \frac{M \sin \alpha}{0.5 + 1.5(a+1)^2} \cdot \frac{4.5}{M \cos \alpha} \cdot y$$

$$z = \frac{9}{1 + 3(a+1)^2} \tan \alpha \cdot y \quad \text{УСЛОВ ПОКЛАПАЮЩА}$$

$$z = \tan(360^\circ - \alpha) y$$

$$\frac{9}{1 + 3(a+1)^2} \tan \alpha = \tan(360^\circ - \alpha)$$

$$\frac{9}{1 + 3(a+1)^2} \tan \alpha = -\tan \alpha$$

$$\frac{9}{1 + 3(a+1)^2} = -1$$

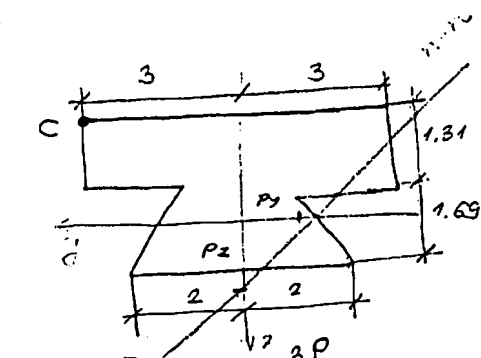
$$1 + 3(a+1)^2 = \frac{9}{-1}$$

$$1 + 3(a+1)^2 = -9$$

$$3(a+1)^2 = -10$$

$$(a+1)^2 = -3.3333$$

1. ЗАДАЧА



$$I_y = 10.21$$

$$I_z = 23.00$$

$$A = 12.00$$

$$C(3, -1.31)$$

P - сила
вращающая

$$G_x = \frac{N}{A} - \frac{I_z}{I_y} y + \frac{I_{yz}}{I_y} z$$

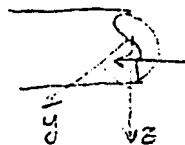
$$M_y = -P e_z$$

$$M_z = P e_y$$

$$G_x = \frac{P}{A} - \frac{P e_y}{I_z} y - \frac{P e_z}{I_y} z \sim \text{ФОРМУЛА}$$

$$P_y = - \frac{I_z z^2}{e_y} = - \frac{I_z}{e_y A}$$

$$P_z = - \frac{I_y y^2}{e_z} = - \frac{I_y}{e_z A}$$



$$G_x = \frac{P}{12.00} - \frac{3P}{23.00} y - \frac{-1.31P}{10.21} z$$

$$G_x = P(0.0833 - 0.1304y + 0.1283z) \sim$$

$$P_y = - \frac{23.00}{3 \cdot 12.00} = -0.6389$$

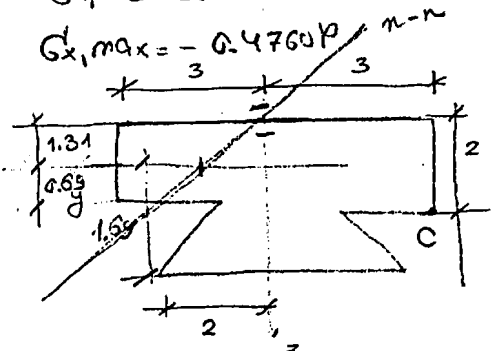
$$P_z = - \frac{10.21}{-1.31 \cdot 12.00} = 0.6495$$

$$G_{x,C} = P(0.0833 - 0.1304 \cdot 3 + 0.1283 \cdot (-1.31))$$

$$G_{x,C} = -0.4750P$$

$$\tau_{max} = \frac{1}{2} |G_{x,max}| = \frac{1}{2} |-0.4760P| = 0.2380P$$

$$G_{x,max} = -0.4760P$$



$$I_y = 10.05$$

$$I_z = 38.50$$

$$A = 15.00$$

$$C(-3, 0.69)$$

~ НОРМАЛЬНИ НАПОН ~

$$G_x = \frac{P}{15.00} - \frac{P(-3)}{38.50} y - \frac{P \cdot 0.69}{10.05} z$$

$$G_x = P(0.0667 + 0.0779y - 0.0686z)$$

~ НЕУТРАЛЬНА ЛИНИЈА ~

$$P_y = - \frac{I_z z^2}{e_y} = - \frac{I_z}{e_y A} = - \frac{38.50}{-3 \cdot 15.00} = 0.8556$$

$$P_z = - \frac{I_y y^2}{e_z} = - \frac{I_y}{e_z A} = - \frac{10.05}{0.69 \cdot 15.00} = -0.9710$$

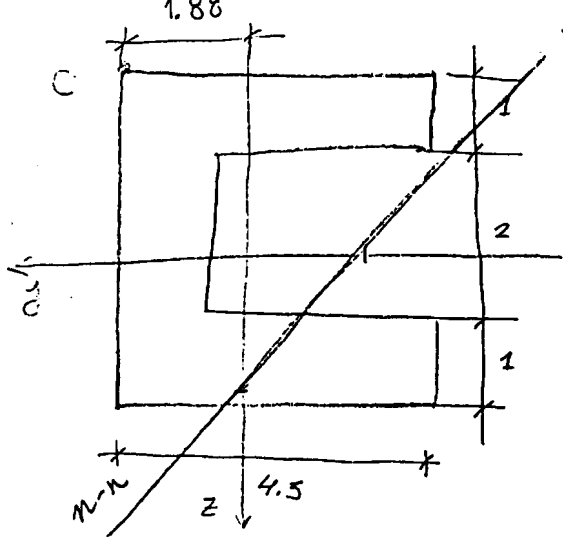
~ МАКСИМАЛНИ СМЕНИЦИ НАПОН ~

$$G_{x,C} = G_{x,max} = P(0.0667 + 0.0779(-3) - 0.0686 \cdot 0.69) = -0.2143P$$

$$\tau_{max} = 0.5 \cdot |G_{x,max}| = 0.5 \cdot |-0.2143P| = 0.1072P$$

4

35 18 35



$$J_z = 20.81$$

$$A = 12.00$$

$$C(1.88, -2)$$

~ НОРМАЛЬНИ НАПЪНИ ~

$$G_x = \frac{P}{12.00} - \frac{P \cdot 1.88}{20.81} y - \frac{P \cdot (-2)}{22.00} z$$

$$G_x = P(0.0833 - 0.0903 y + 0.0909 z)$$

~ МАКСИМАЛНИ СМЪЧУБНИ НАПЪНИ ~

$$\tau_{max} = 0.5 |G_{x, max}| = 0.5 |G_{x, C}|$$

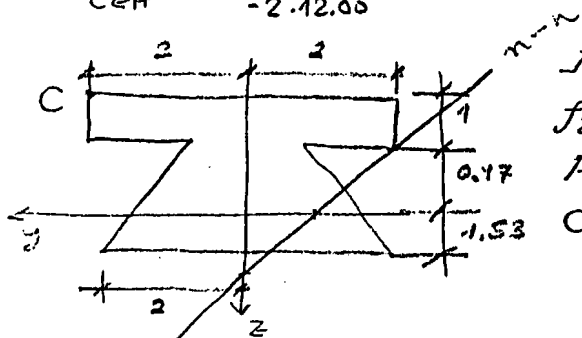
$$G_{x, C} = P(0.0833 - 0.0903 \cdot 1.88 - 0.0909 \cdot 2)$$

$$G_{x, C} = -0.2683 P$$

$$\tau_{max} = 0.5 \cdot |-0.2683 P| = 0.1342 P$$

$$P_y = - \frac{J_z}{e_{yA}} = - \frac{20.81}{1.88 \cdot 12.00} = -0.9224$$

$$P_z = - \frac{J_y}{e_{zA}} = - \frac{22.00}{-2 \cdot 12.00} = 0.9167$$



$$J_y = 8.49$$

$$J_z = 10.33$$

$$A = 10.00$$

$$C(2, -1.47)$$

~ НОРМАЛЬНИ НАПЪНИ ~

$$G_x = P \left(\frac{1}{10.00} - \frac{2}{10.33} y - \frac{-1.47}{8.49} z \right)$$

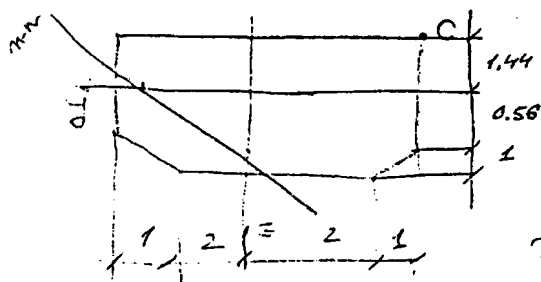
$$P_y = - \frac{J_z}{e_{yA}} = - \frac{10.33}{2 \cdot 10.00} = -0.5165$$

$$G_x = P(0.1 - 0.1936 y + 0.1731 z)$$

$$P_z = - \frac{8.49}{-1.47 \cdot 10.00} = +0.5776$$

$$\tau_{max} = 0.5 \cdot |P \cdot (0.1 - 0.1936 \cdot 2 - 1.47 \cdot 0.1731)|$$

$$\tau_{max} = 0.2708 P$$



$$J_y = 12.00$$

$$J_z = 46.83$$

$$A = 17.00$$

$$C(-3, -1.44)$$

~ НОРМАЛЬНИ НАПЪНИ ~

$$G_x = P \left(\frac{1}{17.00} - \frac{-3}{46.83} y - \frac{-1.44}{12.00} z \right)$$

$$G_x = P(0.0588 + 0.0641 y + 0.12 z)$$

~ МАКСИМАЛНИ СМЪЧУБНИ НАПЪНИ ~

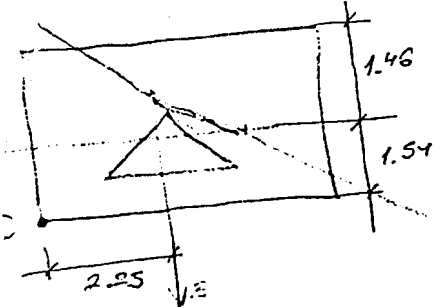
$$\tau_{max} = 0.5 \cdot |P(0.0588 + 0.0641 \cdot (-3) + 0.12 \cdot (-1.44))|$$

$$\tau_{max} = 0.1532 P$$

$$P_y = - \frac{J_z}{e_{yA}} = - \frac{46.83}{17.00 \cdot (-3)} = 0.9182$$

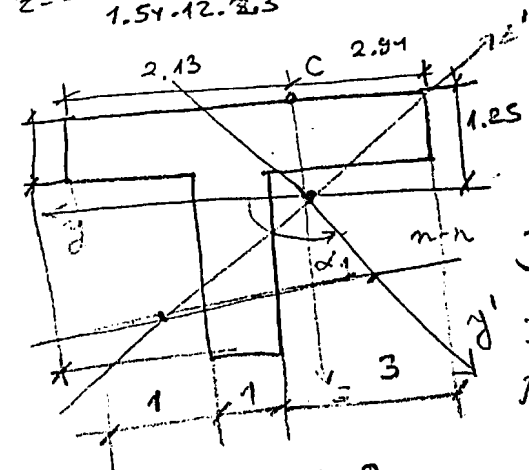
$$P_z = - \frac{12.00}{17.00 \cdot (-1.44)} = 0.4902$$

(2)



$f_z = 22.51$ $G_x = 12.23$ 22.51
 $A = 12.23$
 $C(2.25, 1.54)$ $G_x = P(0.0818 - 0.0001y - 0.1565z)$
 $\sim \text{МАКСИМУМ}$
 $\tau_{\max} = 0.5 \cdot |P(0.0818 - 0.0001 \cdot 2.25 - 0.1565 \cdot 1.54)|$
 $\tau_{\max} = 0.0797P$

$\tau_{\max} = 0.0797P$
 $\tau = -\frac{22.51}{2.25 - 12.23} = -0.8180$
 $\tau = -\frac{9.84}{1.54 - 12.23} = -0.5224$



$f_y = 10.5677$
 $f_z = 12.5417$
 $f_y z = 3.75$

$t g \alpha_{1/2} = \frac{-2 \cdot 3.75}{10.5677 - 12.5417}$
 $A = 8$ $C(0, -1.25)$

$f_{1/2} = \frac{10.5677 + 12.5417}{2} \pm \sqrt{\left(\frac{10.5677 - 12.5417}{2}\right)^2 + 3.75^2}$

$f_1 = 15.4324$ $\alpha_1 = 127.6057^\circ$ $f_y' = 15.4324$
 $f_2 = 7.6770$ $\alpha_2 = 217.6057^\circ$ $f_z' = 7.6770$

$C = \begin{cases} y' = 0.00127.6057^\circ - 1.25 \sin 127.6057^\circ \\ z' = -0.00127.6057^\circ - 1.25 \cos 127.6057^\circ \end{cases}$

$C'(-0.9903; 0.7628)$

$G_x = P\left(\frac{1}{8} - \frac{-0.9903}{7.6770}y - \frac{0.7628}{15.4324}z\right)$

$\sim \text{МАКСИМУМ}$
 $\tau_{\max} = 0.3 \cdot |P(0.125 + 0.1290 \cdot (-0.9903) - 0.0154 \cdot 0.7628)|$
 $\tau_{\max} = 0.0202P$

$G_x = P(0.125 + 0.1290y - 0.0494z)$

МАКСИМУМ

$P_y = -\frac{7.6770}{-0.9903 \cdot 8} = +0.9690$

$P_z = -\frac{15.4324}{0.7628 \cdot 8} = -2.5289$

07

37 19 37

$f_y z = 1.286$ $f_z = f_z' = 3.2550$ $f_1 = f_y' = 6.1969$ $\alpha_1 = 149.5197^\circ$ $\alpha_2 = 239.5197^\circ$
 $\tan \alpha_{1/2} = \frac{-2 \cdot 1.286}{5.44 - 1.012} \quad A = 7.11$

$C(1.21; -1.36)$
 $C' = \begin{cases} y'_C = 1.21 \cos 149.5197^\circ - 1.36 \sin 149.5197^\circ \\ z'_C = -1.21 \sin 149.5197^\circ - 1.36 \cos 149.5197^\circ \end{cases}$
 $C'(-1.7326; 0.5582)$

~ НОРМАЛЬНЫЙ НАПРАВЛЕНИЕ

$$G_x = P \left(\frac{1}{7.00} - \frac{-1.7326}{3.2550} y - \frac{0.5582}{6.1969} z \right)$$

$$G_x = P(0.1428 + 0.5323y + 0.0901z)$$

~ НЕУРАВНОВЕШЕНАЯ ЛИЧНОСТЬ

$$P_y = -\frac{3.2550}{-1.7326 \cdot 7.00} = 0.2684 \quad P_z = -\frac{6.1969}{0.5582 \cdot 7.0} = -1.5859$$

~ МАКСИМАЛЬНЫЕ СМЫЧУЖИ НАПРАВЛЕНИЕ

$$\tau_{max} = 0.5 \cdot P(0.1428 + 0.5323(-1.7326) - 0.0901 \cdot 0.5582)$$

$\tau_{max} = 0.4149 P$
 $f_y = 12.5417$ $f_z = 10.5667$ $f_1 = f_y' = 17.4215$ $\alpha_1 = 37.6235^\circ$ $\alpha_2 = 127.6235^\circ$
 $f_{yz} = -3.75$ $C(0; -1)$ $P_2 = f_z' = 7.6764$
 $\tan \alpha_{1/2} = \frac{-2 \cdot (-3.75)}{12.5417 - 10.5667} \quad A = 8$
 $C' = \begin{cases} y'_C = 0 \cdot \cos 37.6235^\circ - 1 \sin 37.6235^\circ = -0.6105 \\ z'_C = -0 \cdot \sin 37.6235^\circ - 1 \cos 37.6235^\circ = -0.7920 \end{cases}$

~ НОРМАЛЬНЫЙ НАПРАВЛЕНИЕ

$$G_x = P \left(\frac{1}{8} - \frac{-0.6105}{7.6764} y - \frac{-0.7920}{17.4215} z \right)$$

$$G_x = P(0.125 + 0.0795y + 0.0455z)$$

~ МАКСИМАЛЬНЫЕ СМЫЧУЖИ НАПРАВЛЕНИЕ

$$\tau_{max} = 0.5 \cdot P(0.125 - 2.0923 \cdot 0.0795 - 1.0766 \cdot 0.0455)$$

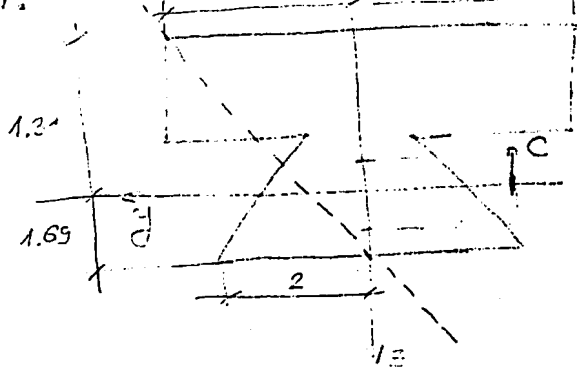
$$\tau_{max} = 0.0452 P$$

$$D(-1; -2.13)$$

$$y'_D = -1 \cos 37.6235^\circ - 2.13 \sin 37.6235^\circ = -2.0923$$

$$z'_D = +1 \sin 37.6235^\circ - 2.13 \cos 37.6235^\circ = -1.0766$$

38



$$F_z = 23.00$$

$$A = 12.00$$

$$P_z = +1.69$$

$$P_y = +1.69$$

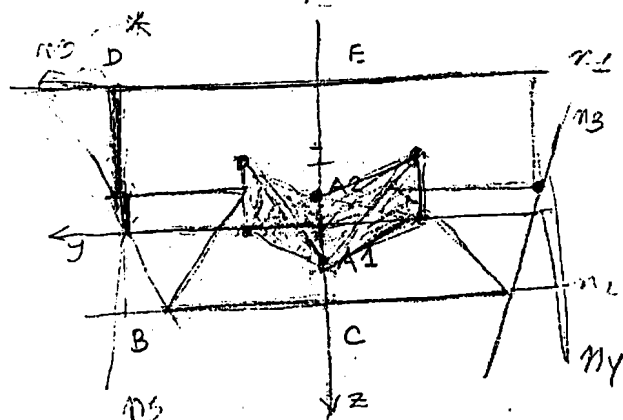
$$x = +1.69$$

$$e_y = - \frac{23.00}{12.00 \cdot 1.69}$$

$$e_y = -1.1341$$

$$e_z = - \frac{10.21}{12.00 \cdot 1.69}$$

$$e_z = -0.5034$$



$$n_1: P_{y1} = \infty$$

$$P_{z1} = -1.31$$

$$A_1(0; 0.6495)$$

$$e_{y1} = - \frac{23.00}{12.00 \cdot \infty} = 0$$

$$e_{z1} = - \frac{10.21}{12.00 \cdot (-1.31)} = 0.6495$$

$$n_2: P_{y2} = \infty$$

$$e_{y2} = 0$$

$$P_{z2} = +1.69$$

$$e_{z2} = - \frac{10.21}{12.00 \cdot 1.69} = -0.5034$$

$$n_3: P_{y3} = 3$$

$$P_{z3} = 5.07$$

$$(1): P_y: P_z = 2: (P_z - 1.69)$$

$$(2): P_y: P_z = (3+1.31): (P_z + 1.31)$$

$$(3): P_y: P_z = x: 1.31$$

$$x = 1.31 \frac{P_y}{P_z}$$

$$P_y (P_z - 1.69) = 2 P_z$$

$$P_y (P_z + 1.31) = P_z - (3 + 1.31 \frac{P_y}{P_z})$$

$$P_y (P_z - 1.69) = 2 P_z$$

$$P_y (P_z + 1.31) = 3 P_z + 1.31 P_y$$

$$P_y P_z - 1.69 P_y = 2 P_z = 0$$

$$P_y P_z + 1.31 P_y - 3 P_z - 1.31 P_y = 0 \quad | \cdot (-1) \quad \uparrow$$

$$P_y - 3 = 0$$

$$P_y = 3$$

$$3 P_z - 1.69 \cdot 3 - 2 P_z = 0$$

$$P_z = 5.07$$

$$e_{y3} = - \frac{23.00}{12.00 \cdot 3} = -0.6333$$

$$e_{z3} = - \frac{10.21}{12.00 \cdot 5.07} = -0.1678$$

$$\frac{P_y}{P_z} = \frac{2}{P_z - 1.69}$$

$$\frac{P_y}{P_z} = \frac{3 + 1.31 \frac{P_y}{P_z}}{P_z + 1.31}$$

$$\frac{2}{P_z - 1.69} = \frac{3 + 1.31 \frac{P_y}{P_z}}{P_z + 1.31}$$

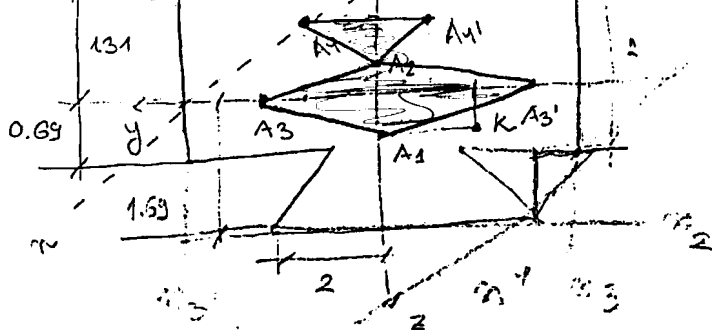
$$2 P_z + 2.62 = 3 P_z - 5.07 + 1.31 P_y - 2.213 \frac{P_y}{P_z}$$

$$-3 P_z$$

$$n_4: P_y = -3$$

$$P_z = \infty \quad e_z = 0$$

$$e_y = - \frac{23.00}{12.00 \cdot (-3)} = 0.6389$$



$$J_z = 38.50$$

$$A = 15.00$$

$$p_y = +3.00$$

$$p_z = -1.31$$

$$e_y = -\frac{38.50}{15.00 \cdot (3.00)} = -0.8556$$

$$e_z = -\frac{10.05}{15.00 \cdot (-1.31)} = +0.5114$$

$$\left. \begin{array}{l} e_y = -0.8556 \\ e_z = +0.5114 \end{array} \right\} K(-0.8556; +0.5114)$$

$$n_1: p_{y1} = \infty \quad e_{y1} = 0$$

$$p_{z1} = -1.31 \quad e_{z1} = -\frac{10.05}{15.00 \cdot (-1.31)} = 0.5114$$

$$\left. \begin{array}{l} e_{y1} = 0 \\ e_{z1} = 0.5114 \end{array} \right\} A_1(0; 0.5114)$$

$$n_2: p_{y2} = \infty \quad e_{y2} = 0$$

$$p_{z2} = 1.69 \quad e_{z2} = -\frac{10.05}{15.00 \cdot 1.69} = -0.3964$$

$$\left. \begin{array}{l} e_{y2} = 0 \\ e_{z2} = -0.3964 \end{array} \right\} A_2(0; -0.3964)$$

$$n_3: p_{y3} = -3 \quad e_{y3} = -\frac{38.50}{15.00 \cdot (-3)} = 0.8556$$

$$p_{z3} = \infty \quad e_{z3} = 0$$

$$\left. \begin{array}{l} e_{y3} = 0.8556 \\ e_{z3} = 0 \end{array} \right\} A_3(0.8556; 0) \Rightarrow A_3'(-0.8556; 0)$$

$$n_4: p_{y4} = -3.69 \quad e_{y4} = -\frac{38.50}{15.00 \cdot (-3.69)} = 0.6956$$

$$p_{z4} = 0.31$$

$$(3-2): (1.69-0.69) = 2 : (p_z - 1.69) \quad (3-2): (1.69-0.69) = (p_y - 2) : 1.69$$

$$1: 1 = 2 : (p_z - 1.69)$$

$$1: 1 = (p_y - 2) : 1.69$$

$$p_z - 1.69 = 2$$

$$1.69 = p_y - 2$$

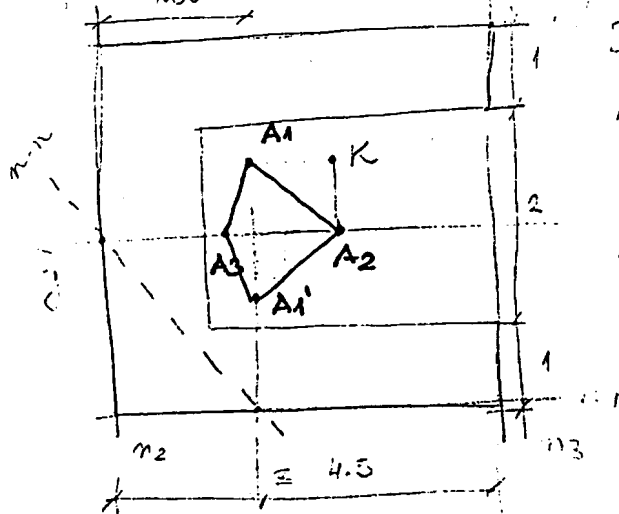
$$p_z = 0.31$$

$$p_y = 3.69$$

$$e_{z4} = -\frac{10.05}{15.00 \cdot (0.31)} = -2.1613$$

$$A_4(0.6956; -2.1613)$$

$$A_4'(-0.6956; -2.1613)$$



$$J_z = 20.81$$

$$A = 12.00$$

$$P_y = 1.88$$

$$P_z = 3$$

$$e_z = - \frac{22.00}{12.00 \cdot 3} = -0.6111$$

$$K(-0.9224; -0.6111)$$

$$n_1: p_{y1} = \infty \quad e_{y1} = 0$$

$$p_{z1} = 3 \quad e_{z1} = - \frac{22.0}{12.0 \cdot 3} = -0.6111$$

$$A_1(0; -0.6111)$$

$$A_1'(0; +0.6111)$$

$$n_2: p_{y2} = 1.88$$

$$p_{z2} = \infty \quad e_{z2} = 0$$

$$e_{y2} = - \frac{20.81}{12.00 \cdot 1.88} = -0.9224$$

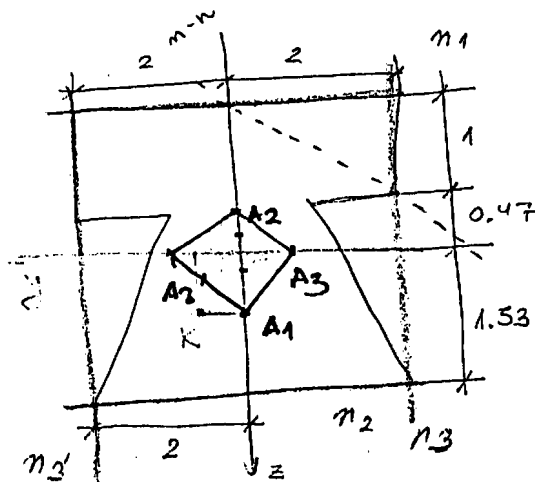
$$A_2(-0.9224; 0)$$

$$n_3: p_{y3} = -(4.5 - 1.88) = -2.62$$

$$p_{z3} = \infty \quad e_{z3} = 0$$

$$e_{y3} = - \frac{20.81}{12.00 \cdot (-2.62)} = 0.6619$$

$$A_3(0.6619; 0)$$



$$J_y = 8.49$$

$$J_z = 10.33$$

$$A = 9.33$$

$$p_{y4} = (p_y - 2) : 0.47$$

$$2 - 0.47 = 1 \cdot (p_y - 2)$$

$$0.94 = p_y - 2$$

$$p_y = -2.94$$

$$p_z = -(1 + 0.47)$$

$$p_z = -1.47$$

$$e_y = - \frac{10.33}{9.33 \cdot (-2.94)} = 0.3766$$

$$e_z = - \frac{8.49}{9.33 \cdot (-1.47)} = 0.6190$$

$$K(0.3766; 0.6190)$$

$$n_1: p_{y1} = \infty$$

$$e_{y1} = 0$$

$$p_{z1} = -1.47$$

$$e_{z1} = - \frac{8.49}{9.33 \cdot (-1.47)} = 0.6190$$

$$A_1(0; 0.6190)$$

$$n_2: p_{y2} = \infty$$

$$e_{y2} = 0$$

$$p_{z2} = 1.53$$

$$e_{z2} = - \frac{8.49}{9.33 \cdot 1.53} = -0.5948$$

$$A_2(0; -0.5948)$$

$$n_3: p_{y3} = 2$$

$$e_{y3} = 0$$

$$p_{z3} = \infty$$

$$e_{y3} = - \frac{10.33}{9.33 \cdot 2} = -0.5536$$

$$A_3(-0.5536; 0)$$

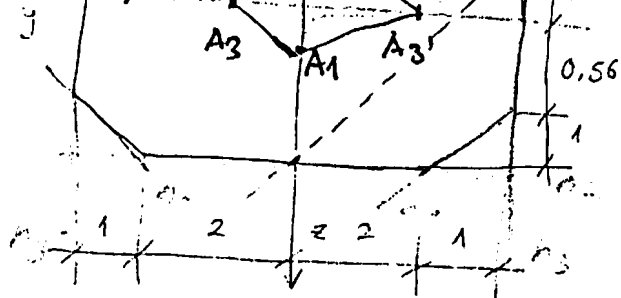
$$A_3'(0.5536; 0)$$

7

7

91

7.1



$$A = 17.00$$

$$p_y = 1.56$$

$$p_z = -1.0833$$

$$p_y = \frac{1.44}{1.44} = 1.0833$$

$$K(2.5429; -0.4525)$$

$$e_y = -\frac{46.83}{17.00 \cdot (-1.0833)} = 2.5429$$

$$e_z = -\frac{12.00}{17.00 \cdot (1.56)} = -0.4525$$

$$n_1: p_{y1} = \infty \quad e_{y1} = 0$$

$$p_{z1} = -1.44 \quad e_{z1} = -\frac{12.00}{17.00 \cdot (-1.44)} = 0.4902 \quad \left. \vphantom{\begin{matrix} p_{z1} \\ e_{z1} \end{matrix}} \right\} A_1(0; 0.4902)$$

$$n_2: p_{y2} = \infty \quad e_{y2} = 0$$

$$p_{z2} = 1.56 \quad e_{z2} = -\frac{12.00}{17.00 \cdot 1.56} = -0.4525 \quad \left. \vphantom{\begin{matrix} p_{z2} \\ e_{z2} \end{matrix}} \right\} A_2(0; -0.4525)$$

$$n_3: p_{y3} = -3 \quad e_{y3} = 0$$

$$p_{z3} = \infty \quad e_{z3} = -\frac{46.83}{17.00 \cdot (-3)} = 0.9182 \quad \left. \vphantom{\begin{matrix} p_{z3} \\ e_{z3} \end{matrix}} \right\} A_3(0.9182; 0); A_3(-0.9182; 0)$$

$$n_4: p_y = 3.56 \quad e_y = -\frac{46.83}{17.00 \cdot (3.56)} = -0.7738$$

$$p_z = 3.56 \quad e_z = -\frac{12.00}{17.00 \cdot (3.56)} = -0.1983 \quad \left. \vphantom{\begin{matrix} e_y \\ e_z \end{matrix}} \right\} A_4(-0.7738; -0.1983)$$

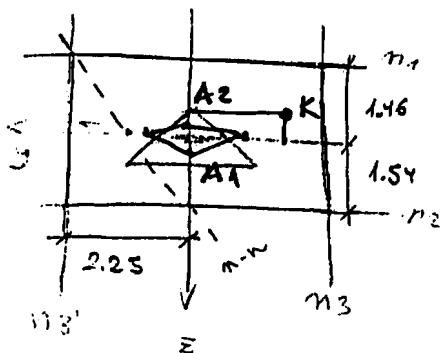
$$p_y = p_z = 1.1$$

$$p_z = 2 \cdot 1 + 0.56 = 3.56$$

$$A_4'(-0.7738; -0.1983)$$

$$p_y = p_z$$

$$e_z = -\frac{12.00}{17.00 \cdot (3.56)} = -0.1983$$



$$p_y = 9.84 \quad p_z = 1.155$$

$$p_z = 22.51 \quad p_z = 1.54$$

$$A = 12.23$$

$$e_y = -\frac{22.51}{12.23 \cdot 1.155} = -1.5936$$

$$e_z = -\frac{9.84}{12.23 \cdot 1.54} = -0.5224 \quad \left. \vphantom{\begin{matrix} e_y \\ e_z \end{matrix}} \right\} K(-1.5936; -0.5224)$$

$$n_1: p_{y1} = \infty \quad p_{y1} = 0 \quad e_{y1} = -\frac{9.84}{12.23 \cdot (-1.46)} = 0.5511$$

$$A_1(0; 0.5511)$$

$$n_2: p_{y2} = \infty \quad e_{y2} = 0 \quad e_{z2} = -\frac{9.84}{12.23 \cdot 1.54} = -0.5224$$

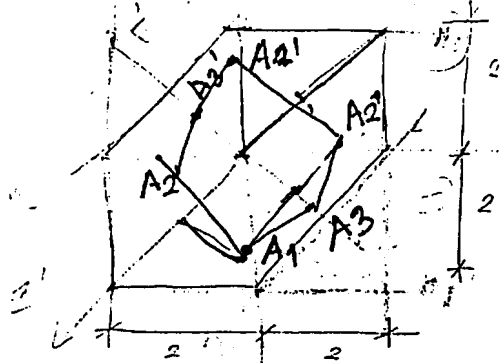
$$A_2(0; -0.5224)$$

$$n_3: p_{y3} = -2.25 \quad e_{y3} = 0$$

$$p_{z3} = \infty \quad e_{z3} = -\frac{22.51}{12.23 \cdot (-2.25)} = 0.8180$$

$$A_3(0.8180; 0)$$

$$A_3'(-0.8180; 0)$$



$$J_3 = \frac{1}{12} \cdot 4^4 - 2 \cdot \frac{1}{36} \cdot \left(\frac{4}{3}\right)^2 \cdot 2 = 21.1358$$

$$J_2 = \frac{1}{12} \cdot 4^4 - 2 \cdot \frac{1}{36} \cdot \left(\frac{4}{3}\right)^2 \cdot 2 = 21.1358$$

$$J_{yz} = 0 + 2 \cdot \frac{1}{72} \cdot \left(\frac{4}{3}\right)^2 \cdot 2 = 0.0988$$

$$J_{1/2} = 21.1358 \pm 0.0988 \quad A = 12$$

$$J_1 = 21.2346 \quad J_2 = 21.037$$

$$\tan 2\alpha_{1/2} = \frac{-2 \cdot 0.0988}{0}$$

$$\alpha_1 = 135^\circ \quad \alpha_2 = 225^\circ$$

$$n_1: p_{y1} = +2\sqrt{2}$$

$$p_{z1} = -2\sqrt{2}$$

$$e_{y1} = -\frac{21.037}{2\sqrt{2} \cdot 12} = -0.6198$$

$$e_{z1} = -\frac{21.2346}{-2\sqrt{2} \cdot 12} = +0.6256$$

$$A_1(-0.6198; +0.6256)$$

$$A_1'(0.6198; +0.6256)$$

$$n_2: p_{y2} = +2\sqrt{2}$$

$$p_{z2} = +2\sqrt{2}$$

$$e_{y2} = -\frac{21.037}{2\sqrt{2} \cdot 12} = -0.6198$$

$$e_{z2} = -\frac{21.2346}{2\sqrt{2} \cdot 12} = -0.6256$$

$$A_2(-0.6198; -0.6256)$$

$$A_2'(0.6198; 0.6256)$$

$$n_3: p_{y3} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

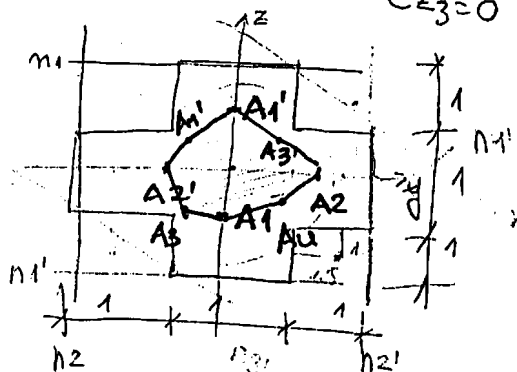
$$p_{z3} = \infty$$

$$e_{y3} = -\frac{21.037}{12\sqrt{2}} = -1.2396$$

$$e_{z3} = 0$$

$$A_3(-1.2396; 0)$$

$$A_3'(1.2396; 0)$$



$$J_y = \frac{1}{12} \cdot 3^4 - 4 \cdot \frac{1}{12} \cdot 1^4 = 6.4167$$

$$J_z = \frac{1}{12} \cdot 3^4 - 4 \cdot \frac{1}{12} \cdot 1^4 = 6.4167$$

$$J_{yz} = 0 \quad A = 3^2 - 4 \cdot 1^2 = 9 - 4 \cdot 1 = 9 - 4 = 5$$

$$n_1: p_{y1} = \infty \quad p_{z1} = 0$$

$$p_{z1} = +1.5 \quad e_{z1} = -\frac{6.4167}{1.5 \cdot 5} = -0.8556$$

$$A_1(0; -0.8556)$$

$$A_1'(0; +0.8556)$$

$$n_2: p_{y2} = -1.5$$

$$e_{y2} = -\frac{6.4167}{-1.5 \cdot 5} = 0.8556$$

$$A_2(0.8556; 0)$$

$$A_2'(-0.8556; 0)$$

$$n_3: p_{y3} = +2$$

$$e_{y3} = -\frac{6.4167}{2 \cdot 5} = -0.6417$$

$$A_3'(-0.6417; -0.6417)$$

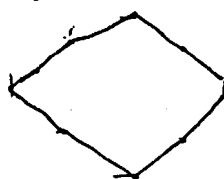
$$p_{z3} = +2$$

$$e_{z3} = -\frac{6.4167}{2 \cdot 5} = -0.6417$$

$$A_3(0.6417; 0.6417)$$

$$A_4(+0.6417; -0.6417)$$

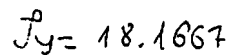
$$A_1(-0.6417; 0.6417)$$



8

9

43 re 43.



$$J_Z = 18.1667$$

$$J_z = -2 \cdot 1.5^2 \cdot 1^2 = -4.5 \quad A = 4^2 - 2 \cdot 1 = 16 - 2 = 14$$

$$J_1 = 22.6667 \quad \alpha_1 = 45^\circ$$

$$f_2 = 13.6667 \quad \alpha_2 = 135^\circ$$

$$J_{1/2} = 18.1667 \pm 4.5$$

$$\eta_1: \begin{matrix} p_{y_1} = 2\sqrt{2} \\ p_{z_1} = 2\sqrt{2} \end{matrix} \quad \left. \begin{matrix} e_{y_1} = -\frac{13.6667}{2\sqrt{2} \cdot 14} = -0.3451 \\ e_{z_1} = -\frac{22.6667}{2\sqrt{2} \cdot 14} = -0.5724 \end{matrix} \right\} \begin{matrix} A_1(-0.3451; -0.5724) \\ A_1'(0.3451; 0.5724) \end{matrix}$$

$$\begin{aligned} n_2: \quad p_{y2} &= 2\sqrt{2} & e_{y2} &= -\frac{13.6667}{2\sqrt{2} \cdot 14} = -0.3451 \\ p_{z2} &= -2\sqrt{2} & e_{z2} &= -\frac{12.6667}{-2\sqrt{2} \cdot 14} = +0.5724 \end{aligned} \quad \left. \begin{array}{l} A_2(-0.3451; 0.5724) \\ A_2'(0.3451; -0.5724) \end{array} \right\}$$

n3: $p_{y3} = -1.5\sqrt{2}$ $e_{y3} = -\frac{13.6667}{-1.5\sqrt{2} \cdot 14} = 0.4602$ $A_3(0.4602; 0)$
 $p_{z3} = \infty$ $e_{z3} = 0$ $A_3'(-0.4602; 0)$

$$J_y = 10.21$$

$$f_z = 23.00$$

$$A = 12.00$$

$B(-1, -1)$

$$p_y = - \frac{23.00}{12.00(-1)} = 1.9167$$

$$p_2 = - \frac{10.21}{12.00(-1)} = 0.8508$$

$$P_y = 10.03$$

$f_z = 38.50$

$$A = 15.00$$

$$B(1; -0.5)$$

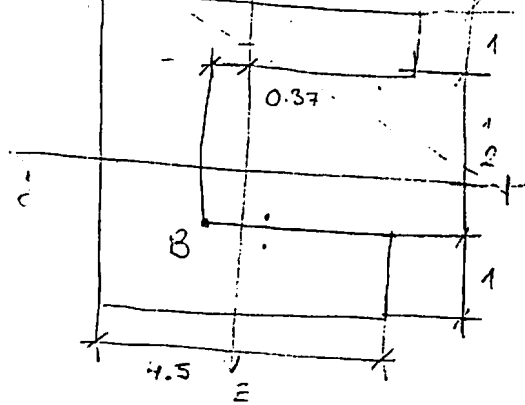
$$p_y = - \frac{38.50}{15.00 \cdot 1} = -2.5667$$

$$p_z = - \frac{10.05}{15.00(-0.3)} = 1.34$$

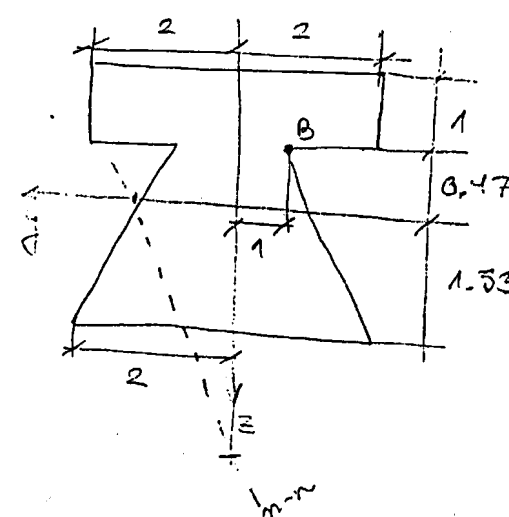
ТРИНА В НЕ ПРИПАДА ЈЕЗГРУ ПРЕСЕКА

10

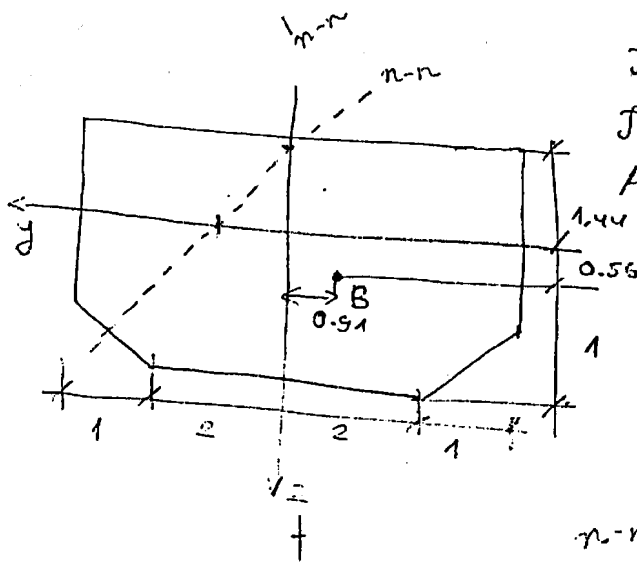
99



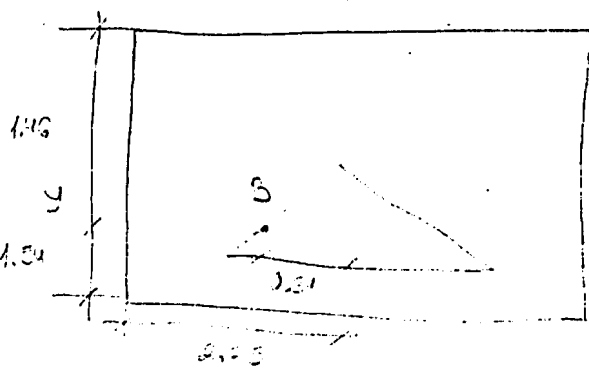
$J_z = 20.87$
 $A = 12.00$
 $B(0.37; 1)$
 $P_z = - \frac{22.00}{12.00 - 1} = -1.8333$
 Точка B не принадлежит жесткой плоскости



$J_y = 8.49$
 $J_z = 10.33$
 $A = 10.00$
 $B(-1; -0.47)$
 $P_y = - \frac{10.33}{12.00 \cdot (-1)} = 1.033$
 $P_z = - \frac{8.49}{12.00(-0.47)} = 1.806$
 Точка B не принадлежит жесткой плоскости



$J_y = 12.00$
 $J_z = 46.83$
 $A = 17.00$
 $B(-0.91; 1.56)$
 $P_y = - \frac{46.83}{17.00 - (-0.91)} = 0.3122$
 $P_z = - \frac{12.00}{17.00(1.56)} = -1.2605$
 Точка B не принадлежит жесткой плоскости



$J_y = 9.84$
 $J_z = 22.51$
 $A = 12.23$
 $B(0.34; 0)$
 $P_y = - \frac{22.51}{12.23 - 0.34} = -5.4134$
 $P_z = - \frac{9.84}{12.23 - 0} = -0.8045$
 Точка B принадлежит жесткой плоскости